Test 3 Jan. 15, 2010

Dynamics Duration 55'

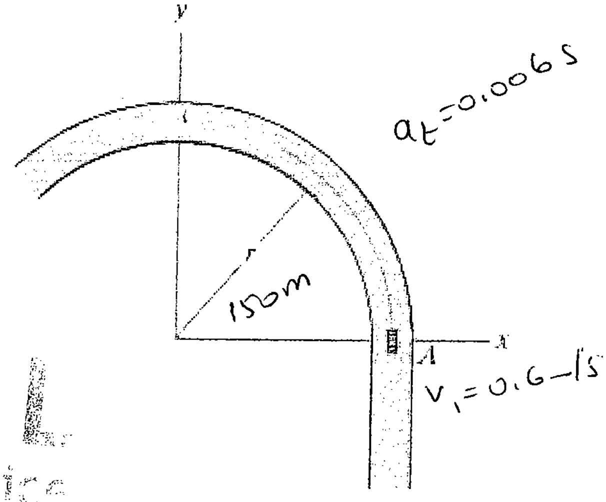
Problem 1 30 Points

A particle has an initial speed V_0=27 m/s. If it experiences a deceleration $a = -6t\ m/s^2$, determine the distance traveled before it stops.

Problem 2 30 Points

The car travels around the portion of a circular track having a radius r = 150m such that when it is at point A it has a velocity V_1 = 0.6 m/s which is increasing at the rate of $\dot{V}$ = 0.006s. Determine the magnitudes of its velocity and acceleration when it has traveled three-fourths of the complete circle.

$\frac{3}{4}(2\pi) = \frac{3\pi}{2}$

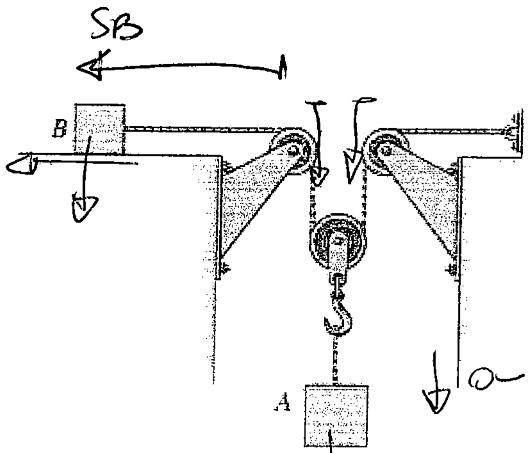


$a_t = 0.006\,s$

$V_1 = 0.6 - 1s$

Problem 3 40 Points

At a given instant block A of weight W_A = 100N (≈ 10kg) is moving downward with speed V_{A0} = 2m/s. Determine its speed at a later time t = 2s. Block B has a weight W_B = 40N (≈ 4 kg) and the coefficient of kinetic friction between it and the horizontal plane is μ_k = 0.2, neglect the mass of the pulleys and cord.



$a\,ds = v\,dv$

$-6t$

$V = -3t^2$

$-3t^2 \Big]_0^3 = S$

$a = \frac{dv}{dt}$

$\int f(t)\,dt = \int dv$

$v = \int_0^t f(t)\,dt$

$v = \frac{ds}{dt}$

$v\,dt = ds$

$S = \int v\,dt$

Problem 1 :

$V_0 = 27\ m/s$

$a = -6t \quad m/s^2$

$V = V_0 + \int 6t\, dt$

$V = 27 + -3t^2$

$a = \dfrac{dv}{dt} \Rightarrow adt = dv$

$\int 27t - 3t^2 dt = \int ds$

$\int_0^t a\,dt = \int_{27}^{0} dV$

$27t - t^3\Big]_0^3 = S$

$S = 54$

$\int_0^t -6t\, dt = \int_{27}^{0} dV$

$-V = -3t^2$

$\Rightarrow V = 3t^2$

$= \dfrac{-6t^2}{2}\Big]_0^t = V\Big]_{27}^{0}$

$-3t^2 = -27$

$t = 3 \Rightarrow t = 3\ sec$

$* \quad v = \dfrac{ds}{dt} \Rightarrow vdt = ds$

$\Rightarrow 3t^2 dt = ds$

$\int_0^3 3t^2 dt = \int_0^s ds$

$t^3\Big]_0^3 = S$

-3

$\boxed{S = 27m}$

Problem 2

$a_t ds = v dv$

$S = \frac{3}{4}(2\pi \times 150)$

$= 225\pi$

$\int a_t ds = \int v dv$

$\int_0^{225\pi} 0.006\, s\, ds = \int_{0.6}^{v} v dv$

$\dfrac{0.006}{2} s^2\Big]_0^{225\pi} = \dfrac{v^2}{2}\Big]_{0.6}^{v}$

$1497.43 = \dfrac{v^2}{2} - \dfrac{0.36}{2}$

$V = 54.73\ m/s$

20 2

USE FOR SCRATCH

$v = \frac{ds}{dt}$

$a\,ds = v\,dv$

$\int_0^5 a\,ds = \int_{27}^{0} v\,dv = \frac{v^2}{2}\Big]_{27}^{0} = 0 - 364.5$

$\int_0^5 (-6t)\,ds = -6ts\Big]_0^5 = -6t(5) = -364.5$

$$a_n = \frac{v^2}{\rho} = \frac{54.73^2}{150} = \cancel{\;} \, m/s^2$$

$$a_t = \dot{v} = 0.006\, s = 0.006\,(225\pi)$$

$$= \cancel{\;}\; 4.24 \; m/s^2$$

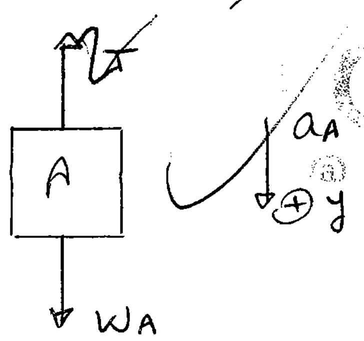
$$a = \sqrt{a_n^2 + a_t^2}$$

$$= \sqrt{(9.97)^2 + (4.24)^2}$$

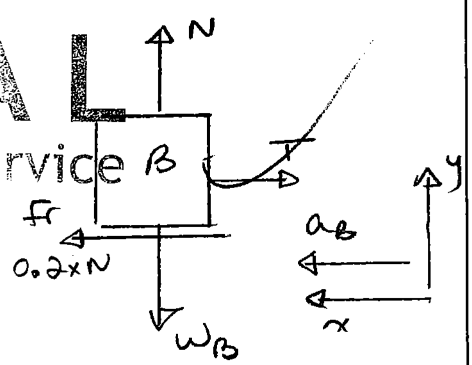
$$= 20.41 \; m/s^2$$

Problem 3

FBD of A (2T)



FBD of B



for mass A:

$$\Sigma F_y = m a_y$$

$$W_A - T = m a_A$$

$$100 - 2T = 10\, a_A \quad --- \; (1)$$

for mass B:

$$\Sigma F_x = m a_x$$

$$0.2 \times N - T = m_B a_B \quad --- \; (2)$$

$$\Sigma F_y = m a_y$$

$$N - W_B = 0 \Rightarrow \boxed{N = 40}$$

replace in (2): $\Rightarrow \; 8 - T = m_B a_B \quad --- \; (2')$

20 4

USE FOR SCRATCH

$* \quad 2S_A + S_B + cst = cst$

$$2\frac{dS_A}{dt} + \frac{dS_B}{dt} + 0 = 0$$

$$2v_A + v_B = 0$$

$$2\frac{dv_A}{dt} + \frac{dv_B}{dt} = 0$$

$$2a_A + a_B = 0 \Rightarrow 2a_A = -a_B \quad \text{--- (3)}$$

Replace in (2')

$$\Rightarrow 8 - T = -2m_B a_A \Rightarrow 8 - T = -8a_A$$

$$\Rightarrow \begin{cases} 2T - 10a_A = -100 \\ -T + 8a_A = -8 \end{cases}$$

Solving : $T = 48.89\ N$

$a_A = 5.11\ m/s^2 \downarrow$

$a_B = -2a_A = -10.22\ m/s^2$

$a_B = 10.22\ m/s^2 \rightarrow$

-7

$V = a_A t + V_0$

$V = 5.11(2) + 2 = 12.22\ m/s$

-3

20 6

USE FOR SCRATCH

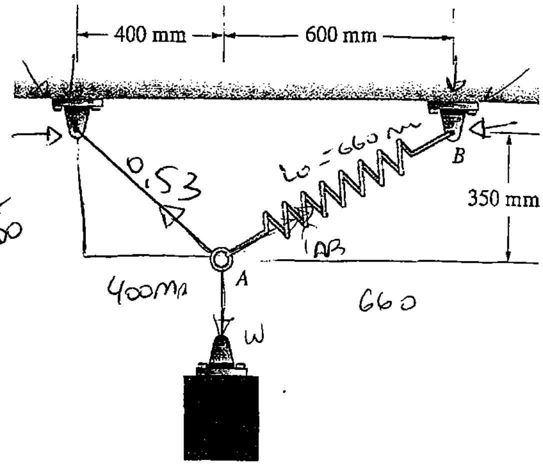
A pt => no moment (only ΣFx & ΣFy)
A load, plank, object —— => (ΣFx, ΣFy + ΣM)

GNE212
Test1

November 20, 2009

Problem 1 (30 points)

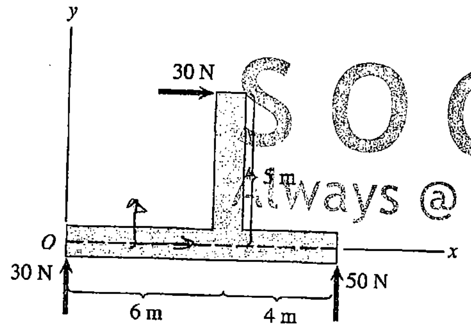
The unstretched length of the spring AB is 660 mm, and the spring constant k = 1000 N/m. What is the mass of the suspended object?



Problem 2 (28 points)

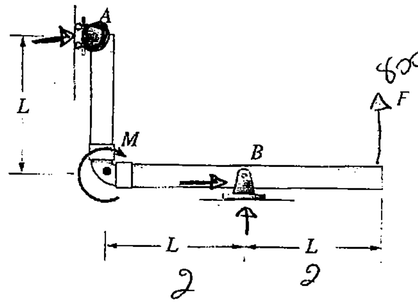
Reduce the system of forces by:

1. A single force F and a couple M at point O.
2. A single force. Where does the line of action of the force intersects the x-axis.



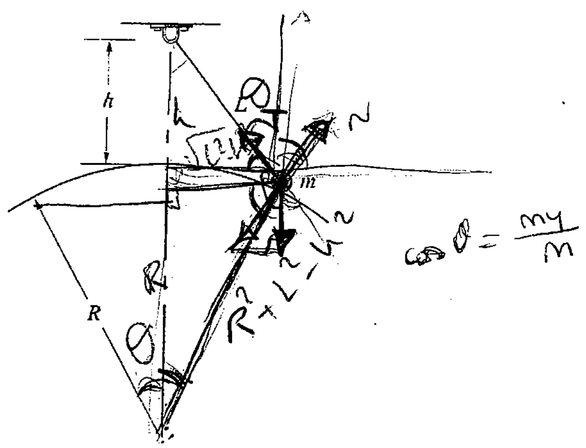
Problem 3 (28 points)

The beam is supported by a roller at A and pin at B. If F = 800 N, M = 200 N-m and L = 2 m. What are the reactions at A and B?



Problem 4 (14 points)

The small sphere of mass m is attached to a string of length L and rest on a smooth surface of a sphere of radius R. Determine the tension in the string in terms of m, L ,h and R.

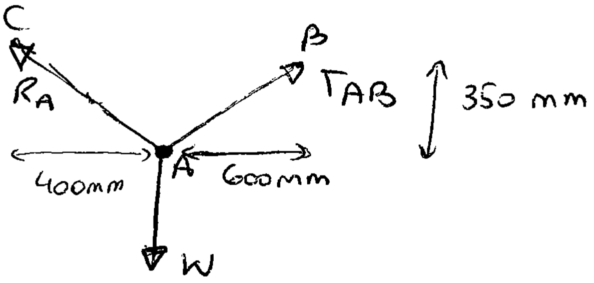


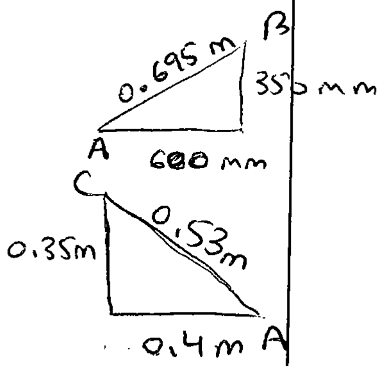
$\cos\theta = \frac{my}{m}$

4
25
9
225

Problem 1

Free body diagram of A.





$\overset{+}{\rightarrow}$ $\sum F_x = 0 \Rightarrow T_{AB}\left(\frac{0.6}{0.695}\right) - \left(\frac{0.4}{0.63}\right) R_A = 0$

$$* \ T_{AB} = k(\Delta L)$$
$$= 1000\,(0.695 - 0.66)$$
$$= 35\ N$$

$$\Rightarrow 35\left(\frac{0.6}{0.695}\right) - R_A\left(\frac{0.4}{0.63}\right) = 0$$

$$\Rightarrow R_A = 47.6\ N$$

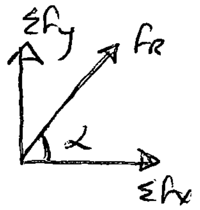
$\overset{+}{\uparrow}$ $\sum F_y = 0 \Rightarrow R_A\left(\frac{0.35}{0.53}\right) + T_{AB}\left(\frac{0.35}{0.695}\right) - W = 0$

$$\Rightarrow (47.6 \times 0.66) + (35 \times 0.5) - W = 0$$

$$\Rightarrow W = 49\ N$$

$$\Rightarrow m = \frac{W}{g} = \frac{49}{9.8} = 5\ kg$$

Problem 2



$\overset{+}{\rightarrow}$ 1. $\sum F_x = 30N \rightarrow$

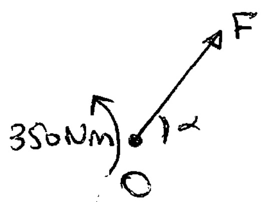
$\overset{+}{\uparrow}$ $\sum F_y = 30N + 50N = 80N \uparrow$

$$\Rightarrow F_R = \sqrt{30^2 + 80^2}$$
$$= 85.44N$$

$$\tan\alpha = \frac{F_y}{F_x} = \frac{80}{30}$$

$$\Rightarrow \alpha = \tan^{-1}\frac{80}{30}$$

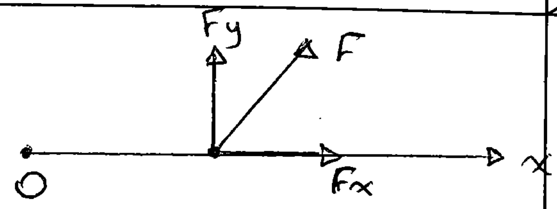
$$\alpha = 69.4^\circ$$



$\overset{+}{\circlearrowleft}$ $\sum M_O = -30(5) + 50(10) = 350\ Nm \circlearrowleft$

2
20

USE FOR SCRATCH



(+)↺ 2. $\Sigma M_O = M_{R(O)}$

$$350 = +F_y(d) + F_x(0)$$

$$350 = 80 \times d$$

$$\Rightarrow d = 4.375 \text{ m}$$

=> The line of action of the force intersects the x-axis at a distance $d = 4.375$ m from pt. O

Problem 3

(+)→ $\Sigma F_x = 0 \Rightarrow R_{Ax} + R_{Bx} = 0$

(+)↑ $\Sigma F_y = 0 \Rightarrow R_{By} + 800 = 0$

$\Rightarrow R_{By} = 800$ N ↓

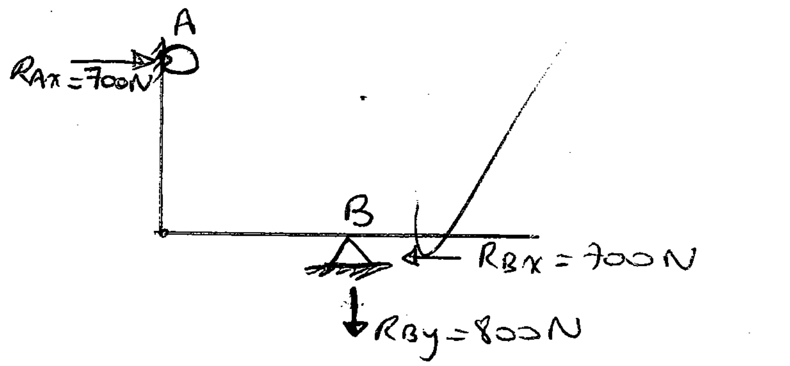
(+)↺ $\Sigma M_B = 0 \Rightarrow -R_{Ax}(2) - 200 + 800(2) = 0$

$\Rightarrow R_{Ax} = 700$ N →

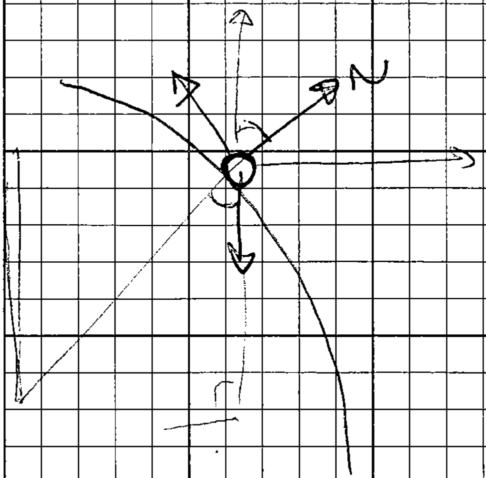
replace in 1st equation, $\Rightarrow R_{Bx} = -700$ N

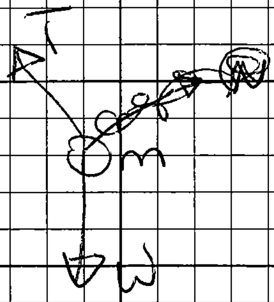
$R_{Bx} = 700$ N ←

=>



USE FOR SCRATCH





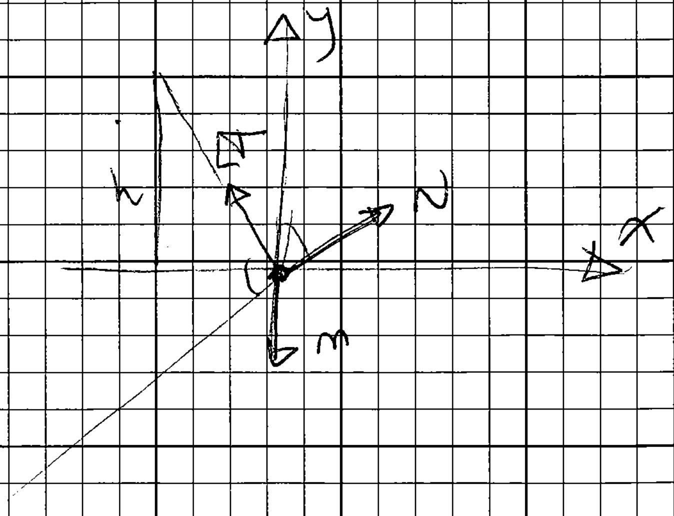
$$\Sigma F_y = 0 \Rightarrow -w + T\left(\frac{L}{h}\right) = 0$$

$$T\left(\frac{L}{h}\right) = w$$

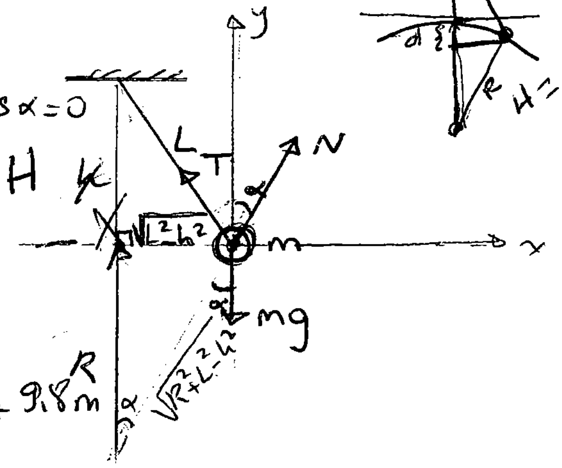
$$T = \frac{wh}{L}$$

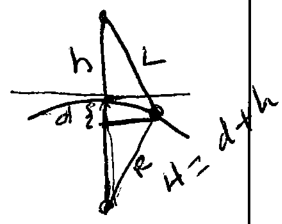
$$= \frac{mgh}{L}$$

$$-w + T\left(\frac{L}{h}\right) + N$$



Problem 4





$$\uparrow + \ \Sigma f_y = 0 \Rightarrow -mg + T\left(\frac{h}{L}\right) + N\cos\alpha = 0$$

$$\cos\alpha = \frac{R}{\sqrt{R^2+L^2-h^2}}$$

$$\Rightarrow T\left(\frac{h}{L}\right) = -N\cos\alpha + mg$$

$$T\left(\frac{h}{L}\right) = -N\left(\frac{R}{\sqrt{R^2+L^2-h^2}}\right) + 9.8m$$

$$\xrightarrow{+} \ \Sigma f_x = 0 \Rightarrow -T\left(\frac{\sqrt{L^2-h^2}}{L}\right) + N\left(\frac{\sqrt{L^2-h^2}}{\sqrt{R^2+L^2-h^2}}\right) = 0$$

$$\Rightarrow N = \frac{T}{L}\left(\sqrt{L^2-h^2}\right) \div \left(\frac{\sqrt{L^2-h^2}}{\sqrt{R^2+L^2-h^2}}\right)$$

$$\Rightarrow N = \frac{T}{L}\cdot\sqrt{R^2+L^2-h^2}$$

$$\Rightarrow T\left(\frac{h}{L}\right) = -\frac{T}{L}\left(\cancel{\sqrt{R^2+L^2-h^2}}\right)\left(\frac{R}{\cancel{\sqrt{R^2+L^2-h^2}}}\right) + 9.8m$$

$$\Rightarrow T\left(\frac{h}{L}\right) = -\frac{T}{L}\cdot R + 9.8m$$

$$T\left(\frac{h}{L}\right) + T\left(\frac{R}{L}\right) = 9.8m$$

$$T\left(\frac{h}{L}+\frac{R}{L}\right) = 9.8m$$

-1

$$\Rightarrow T = \frac{9.8\,mL}{h+R}$$

20 6

USE FOR SCRATCH

take +ve acc to one assumed
If answer is -ve ⇒ opp to assumed direction

LEBANESE AMERICAN UNIVERSITY الجامعة اللبنانية الأميركية

EXAMINATION BOOKLET

80/100

Name [redacted]
Subject Mechanics
Instructor Dr. Habib Rai

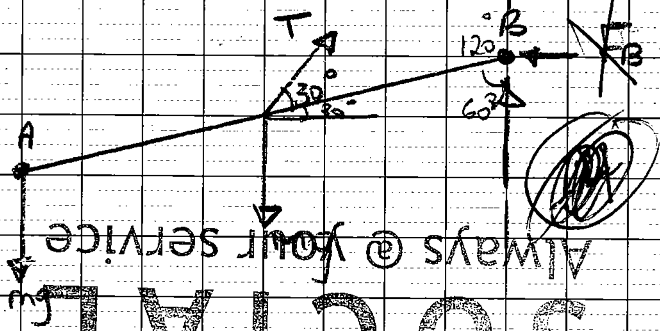
Date 21-12-2009
I.D. No. [redacted]
Section
Box No.

(Begin here and write on both sides)

1. 16
2. 39
3. 25

80

Problem 1



Applying equilibrium:

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow -F_B + T\cos 60^\circ = 0$

$\overset{+}{\uparrow} \Sigma F_y = 0 \Rightarrow -mg - Mg + T\sin 60^\circ = 0$

$-(200)(9.81) - 50(9.81) + T\sin 60^\circ = 0$

$-1962 - 490.5 + 0.866T = 0$

$\Rightarrow T = 2831.9\ N$

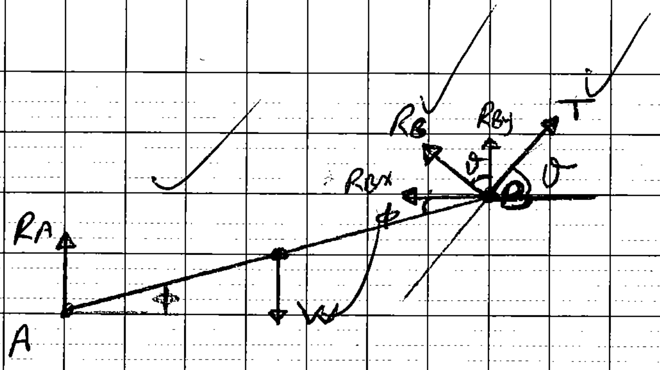
$\Rightarrow F_B = T\cos 60^\circ$

$= 2831.9 \cos 60^\circ$

$\Rightarrow 1415.95\ N$

-14

Problem 3



Applying equilibrium:

$$\overset{+}{\rightarrow}\ \Sigma F_x = 0 \Rightarrow -R_{Bx} + T\cos\theta = 0$$

$$-R_B\sin\theta + T\cos\theta = 0$$

$$R_B\sin\theta = T\cos\theta$$

$$R_B = T\cot\theta$$

$$\overset{+}{\circlearrowleft}\ \Sigma M_B = 0 \Rightarrow -R_A(L\cos\phi) + W\left(\tfrac{L}{2}\cos\phi\right) = 0$$

$$R_A \cdot L\cos\phi = W \cdot \frac{L}{2}\cos\phi$$

$$R_A = \frac{W}{2}$$

$$\overset{+}{\uparrow}\ \Sigma F_y = 0 \Rightarrow R_A - W + T\sin\theta + R_B\cos\theta = 0$$

$$\frac{W}{2} - W + T\sin\theta + T\cot\theta\cos\theta = 0$$

$$-\frac{W}{2} + T(\sin\theta + \cot\theta\cos\theta) = 0$$

$$T\left(\sin\theta + \frac{\cos\theta\cos\theta}{\sin\theta}\right) = \frac{W}{2}$$

$$T\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}\right) = \frac{W}{2}$$

$$T\left(\frac{1}{\sin\theta}\right) = \frac{W}{2}$$

$$\boxed{T = \frac{W}{2}\sin\theta}$$

Dynamics Homework

Problem 1

$V_0 = 70\,km/h = \frac{70\times10^3}{3600} = 19.44\ m/s$

$a = \frac{6000\,km}{hr^2} = \frac{6000\times10^3}{(3600)^2} = 0.46\ m/s^2$

- $V = V_0 + at \Rightarrow V - V_0 = at$

$V_f = 120\,km/hr = \frac{120\times10^3}{3600} = 33.33\ m/s$

$\Rightarrow 33.33 - 19.44 = 0.46t \Rightarrow t = 30.2\ sec$

- $S = \frac{1}{2}at^2 + V_0 t + \cancel{S_0}^{\,0}$

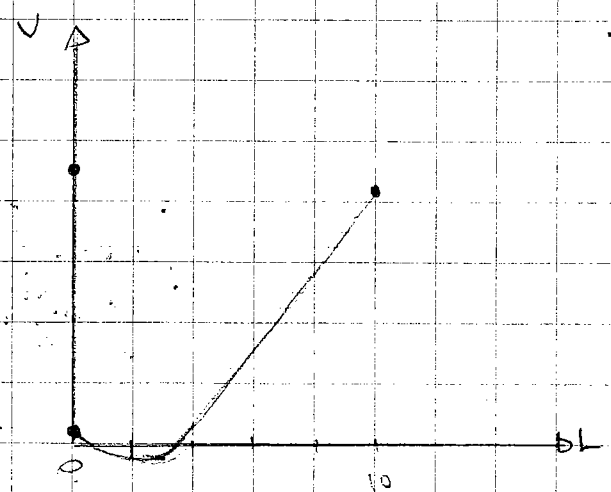
$= \frac{1}{2}(0.46)(30.2)^2 + 19.44(30.2) = 796.86\ m$

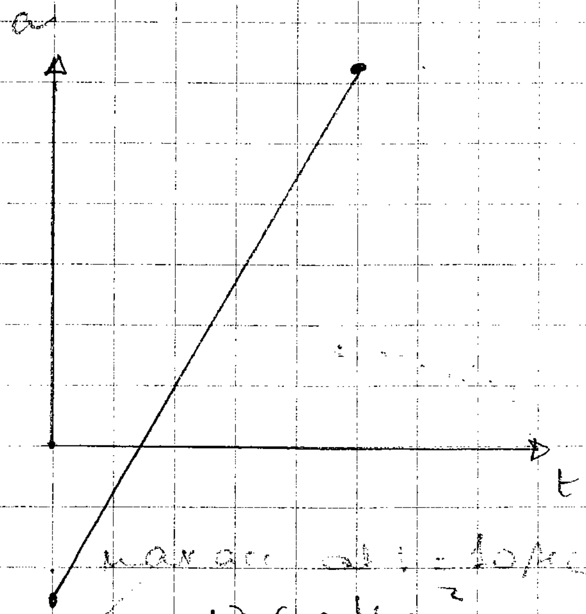
Problem 2

$S = 0.3t^3 - 2.7t^2 + 4.5t$

$V = 0.9t^2 - 5.4t + 4.5$

$a = 1.8t - 5.4$





$\Rightarrow$ max velocity at $t = 10$ sec

$V = 40.5\ m/sec$

max acc at $t = 10$ sec

$a = 12.6\ m/sec^2$

Problem 3

* $a = (-2v)\ m/s^2$

$t=0,\ s=0,\ v_0 = 20\,m/s$

$v\,dv = a\,ds \Rightarrow v\,dv = -2v\,ds$

$\int v\,dv = \int (-2v)\,ds$

$\dfrac{v\,dv}{-2v} = ds \Rightarrow \dfrac{dv}{-2} = ds$

put the common together, the "v" with the "v" & the "s" with the "s"

$-\frac{1}{2}\int_{20}^{v} dv = \int_0^s ds$

$-\frac{1}{2}[v-20] = s$

$-\frac{1}{2}v + 10 = s$

$-\frac{1}{2}v = s - 10$

$\boxed{v = -2s + 20}$

* position stops when $v = 0$

$\Rightarrow 0 = -2s + 20$

$2s = 20 \Rightarrow s = 10\,m.$

Problem 4

$t=0,\ s_0 = 1m,\ v_0 = 2\,m/s,\ a = (2t-1)\ m/s^2$

$a = \dfrac{dv}{dt} \Rightarrow dv = a\,dt \Rightarrow dv = (2t-1)\,dt$

$\int_2^v dv = \int_0^6 (2t-1)\,dt$

$v = t^2 - t + 2$

$v - 2 = t^2 - t\Big]_0^6 = 36 - 6 = 30$

$v = 32\ m/s$

$\int_0^6 v\,dt = \int_1^s ds$

$\dfrac{t^3}{3} - \dfrac{t^2}{2} + 2t\Big]_0^6 = s - 1$

$\Rightarrow s = 67m$

$\Rightarrow \Delta s = 66m$

* ~~$v = \dfrac{ds}{dt} \Rightarrow v\,dt = ds \Rightarrow \int_0^6 32\,dt = \int_1^s ds$~~

~~$32 \times 6 = s - 1 \Rightarrow s = 193\ m$~~

* ~~total distance, $\Delta s = s - s_0 = 193 - 1 = 192\ m$~~

②

~~Problem 5~~

$t=0, V_0=0, S_0=0$

- $a_A = 6t-3$; $a_B = 12t^2-8$; at $t=4$?

$a\,dt = dv$

$\Rightarrow \int_0^4 (6t-3)dt = \int_0^v dv$ $\int_0^4 (12t^2-8)dt = \int_0^v dv$

$\Rightarrow 3t^2 - 3t\Big]_0^4 = V$ $\Rightarrow 4t^3 - 8t\Big]_0^4 = V$

$48 - 12 = V \Rightarrow V_a = 36\, m/s$ $256 - 32 = V \Rightarrow V_b = 224\, m/s$

$v\,dt = ds$

$\Rightarrow \int_0^4 36\,dt = \int_0^s ds$ $\int_0^4 224\,dt = \int_0^s ds$

$\Rightarrow 36\times 4 = S_a = 144\, m$ $224\times 4 = S = 896\, m$

$\Rightarrow \Delta s = 752$

Problem 6:

Ball A: $V_0 = 0$; $S_0 = 12m$, $S_f = 6m$

Ball B: $V_0 = ?$; $V_f = ?$; $S_0 = 1.5$; $S_f = 6m$

Ball A: $S_a = \frac{1}{2}at^2 + \cancel{V_0 t} + S_0$

$S_a = \frac{1}{2}at^2 + 12$

$V = at + \cancel{V_0}$

$V^2 = \cancel{V_0^2} + 2as$

$at^2 = 2a(-6)$

$at^2 = -12$

$\Rightarrow$

Ball B: $S = \frac{1}{2}at^2 + V_0 t + S_0$

$6 = \frac{1}{2}at^2 + V_0 t + 1.5$

$4.5 = \frac{1}{2}at^2 + V_0 t$

- $V^2 = V_0^2 + 2a(4.5)$

$V^2 = V_0^2 + 9a$

$V^2 - V_0^2 = 9a$

- $V = at + V_0$

$\frac{1}{2}at^2 + 6 = \frac{1}{2}at^2 + V_0 t + 6$

$\frac{1}{2}at = \frac{1}{2}at + V_0$

$-6 - \frac{1}{2}at^2 = (V^2 - 9a)t$

Problem 7

- $V = 5\,m/s$ (constant)
- $S: y = x^2 - 4$
- constant speed $\Rightarrow \dot{v} = 0 \Rightarrow a_t = 0$

$$\Rightarrow a = a_n = \frac{v^2}{\rho} = v^2 \cdot \frac{1}{\rho}$$

$$* \frac{1}{\rho} = \frac{y''}{(1+y'^2)^{3/2}} = \frac{2}{(1+(2x)^2)^{3/2}}$$

$$= \frac{2}{(1+4x^2)^{3/2}}$$

- $y = x^2 - 4$
- $y' = 2x$
- $y'' = 2$

$$\Rightarrow a = a_n = 25 \cdot \frac{2}{(1+4x^2)^{3/2}} = \frac{50}{(1+4x^2)^{3/2}}$$

$$\underline{\text{deriv}} = - \frac{3/2\,(1+4x^2)^{1/2}(8x)(50)}{(1+4x^2)^3}$$

$$= \frac{-600x\sqrt{1+4x^2}}{(1+4x^2)^3} \longrightarrow \text{always +ve}$$

$$\Rightarrow \text{deriv} = 0 \Rightarrow x\underbrace{\sqrt{1+4x^2}}_{\text{always +ve}} = 0$$

$$\Rightarrow \boxed{x = 0}$$

$\Rightarrow$ at $x = 0$, $y = -4 \Rightarrow$ pt. $(0, -4)$, the mag of the acceleration is max.

$$a = a_n = \frac{50}{1} = 50\ m/s^2$$

Problem 8

$$a_t = a\cos\theta = 14\cos 75^\circ = 3.62\ m/s^2$$
$$a_n = a\sin\theta = 14\sin 75^\circ = 13.52\ m/s^2$$

$$* \ a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{20^2}{13.52} = 29.59\ m$$

(3)

Problem 10:

$a_t = 0.05s$

$r = 50\ m$; $V_0 = 4\ m/s$

$S_0 = 0$ $S_f = 10\ m$ $\Rightarrow \Delta s = 10\ m$

* $a\,ds = v\,dv$

$\Rightarrow \int_0^{10} 0.05s\,ds = \int_4^{v} v\,dv$

$\frac{0.05}{2}\left[s^2\right]_0^{10} = \frac{1}{2}v^2\Big]_4^{v}$

$2.5 = \frac{1}{2}[v^2 - 16]$

$5 = v^2 - 16 \Rightarrow v^2 = 21 \Rightarrow \boxed{v = 4.58\ m/s}$

* $a_t = 0.05s = 0.05(10) = 0.5\ m/s^2$

$a_n = \frac{v^2}{\rho} = \frac{v^2}{r} = \frac{4.58^2}{50} = 0.42\ m/s^2$

$\Rightarrow a = \sqrt{a_n^2 + a_t^2} = 0.65\ m/s^2$

Problem 11:

~~Problem~~ 5

$\int dv = \int a\,dt$

$\int_0^v dv = \int_0^4 (6t - 3)\,dt$

$V = 3t^2 - 3t$

$\int v\,dt = \int ds$

$\int (3t^2 - 3t)\,dt = \int_0^s ds$

$t^3 - \frac{3}{2}t^2 = S$

$\boxed{at}$: $t = 4 \implies 4^3 - 24 = S$

$\implies S = 40m$

$\int dv = \int a\,dt$

$V = 12t^2 - 8$

~~$\int v\,dt$~~ $\int_0^t (12t^2 - 8)\,dt = \int_0^v dv$

$4t^3 - 8t = v$

$\implies \int v\,dt = \int ds$

$\int (4t^3 - 8t)\,dt = \int_0^s ds$

$t^4 - 4t^2 = S$

$4^4 - 64 = S$

$256 - 64 = S$

$\implies S = 192m$

2
16
4
64

24
16
4
64
4
256
64
192

Dynamics

Homework

Problem 1

Traveling with an initial speed *of 70km/h,* a car accelerates at 6000km/hr^2 along a straight road. How long will it take to reach a speed of *120km/h*? Also, through what distance does the car travel during this time?

$S = 792 m$

Problem 2

The position of a particle along a straight line is given by $s = (0.3t3 + -2.7t2 + 4.5t)$ where t is in seconds. Determine its maximum acceleration and maximum velocity during the time interval $0 \le t \le 10$s.

$a = 12.6 m/s^2$

$v = 40.5 m/s$

Problem 3

A particle is moving along a straight line such that its acceleration is defined as a = (-2v) m/s^2, where v is in meter per seconds .If v =20m/s when s = 0 and t = 0, determine the particle's velocity as a function of position and the distance the particle moves before it stops.

acc function ⇒ separable of the 3 integrals

$S = 10 m$

Problem 4

The acceleration of a particle as it moves along a straight line is given by a = (2t -1) m/s^2, where t is in seconds. If $s = 1m$ *and* v =2m/s *when* t = 0, determine the particle's velocity and position when $t = 6s$. Also, determine the total distance the particle travels during this time period.

$v = 32 m/s$

Problem 5

Two particles A and B start from rest at the origin s = 0 and move along a straight line such that a_A = $(6t\text{-}3)$ and $a_B = (12t2\text{-}8)$, where t is in seconds. Determine the distance between them at t=4s and the total distance each has traveled in time t=4s.

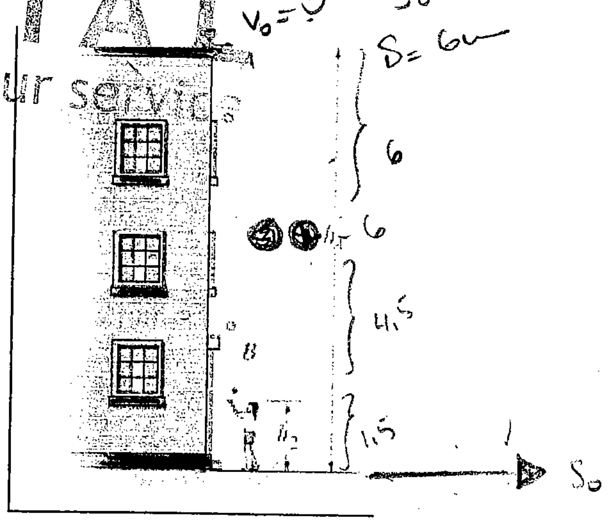
$S_A = 40m$ $S_B = 192 m$

Problem 6

Ball A is released from rest at a height of 12m at the same time that a second ball B is thrown upward 1.5m from the ground. If the balls pass one another at a height of 6m, determine the speed at which ball B was thrown upward..

same time $V_B = 9.5 m/s$

$V_0 = 0$ $S_0 = 12m$ $S = 6m$



Problem 7

A particle P moves along the curve $y = (x2 \text{-}4)$ with a constant speed *of 5m/s*. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

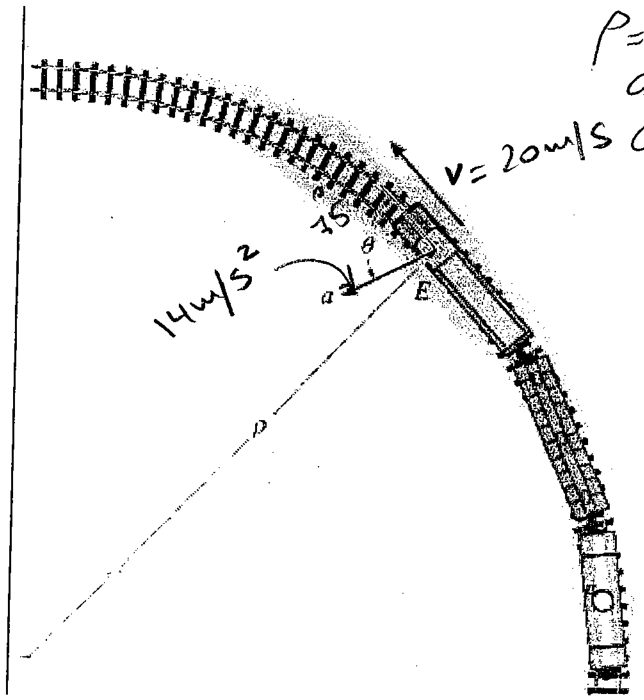
$a = 50 m/s^2$

at

Problem 8

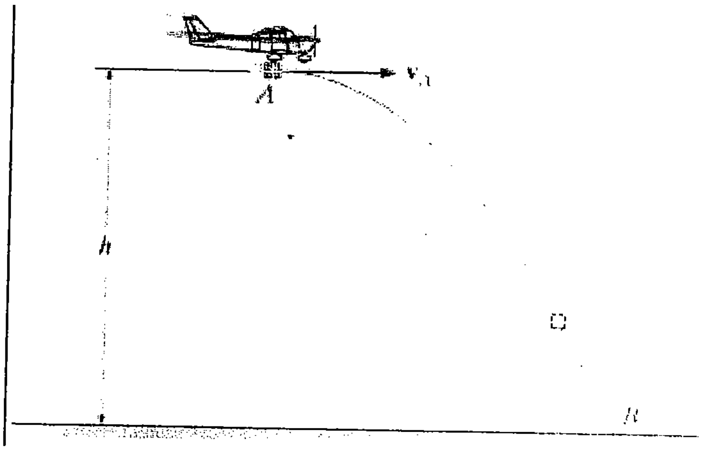
At a given instant the train engine at E has a speed of $20 m/s$ and an acceleration of $14 m/s^2$ acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path. The angle between a and v is equal to 75 degree

$\rho = 29.58$ m
$a_T = 3.62$ m/s^2
$v = 20$ m/s $a_N = 13.563$ m/s^2



Problem 9 XX

A package is dropped from the plane which is flying with a constant horizontal velocity $v_A = 50 m/s$. Determine the normal and tangential components of acceleration and the radius of curvature of the path of motion (a) at the moment the package is released at A, where it has a horizontal velocity $v_A = 50 m/s$, and (b) *just before* it strikes the ground at B. With h =500m.

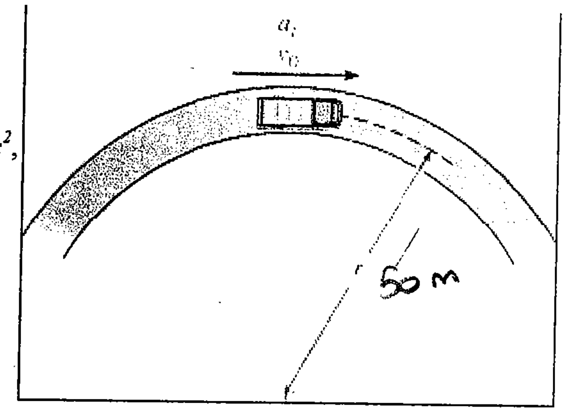


DAWAA

$V = 4 m/s$

Problem 10

The truck travels in a circular path having a radius Of 50m at a speed of 4m/s. For a short distance from $s = 0$, its speed is increased by $a_t = (0.05s)\ m/s^2$, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved a distance $s = 10$m.
Given r=50m



$a_n = 0.419$

$v = 4.58\ m/s$

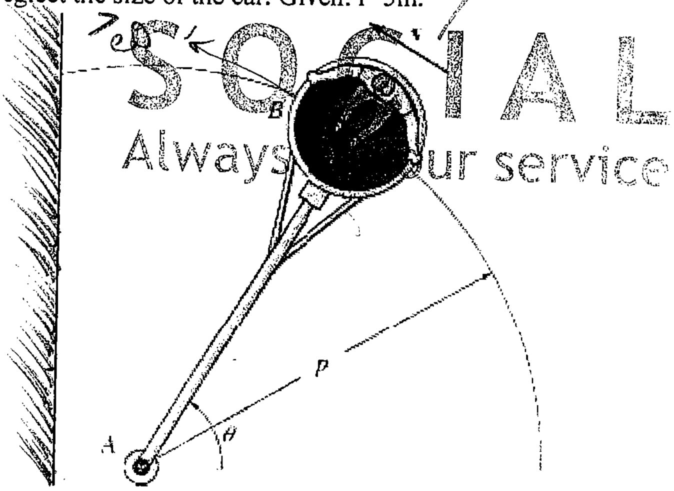
$a = 0.652\ m/s^2$

Problem 11

The car *B* turns such that its speed is increased by $dv_B/dt = (0.5e^t) m/s^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm *AB* rotates to $\theta = 30°$. Neglect the size of the car. Given: r=5m.

$t = 2.123\ s$

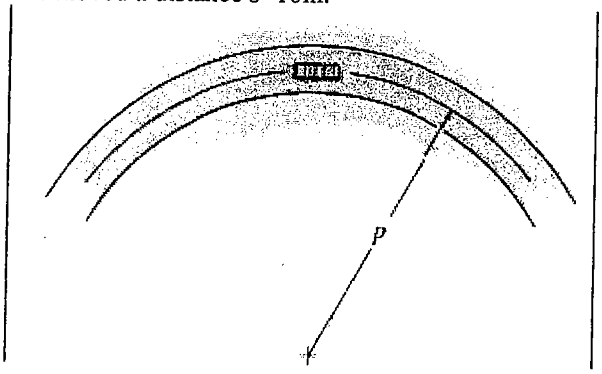
$V = 4.97\ m/s$



same of 10

Problem 12

The truck travels at speed of 4m/s along a circular road that has radius of 50m. For a short distance from $s = 0$, its speed is then increased by $dv/dt = (0.05s) m/s^2$. Determine its speed and the magnitude of its acceleration when it has moved a distance s=10m.



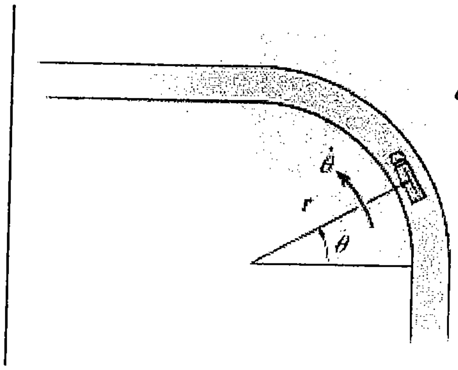
$a_t = 0.5\ m/s^2$

$a_n = 0.419\ m/s$

No polar coordinates in test

Problem 13

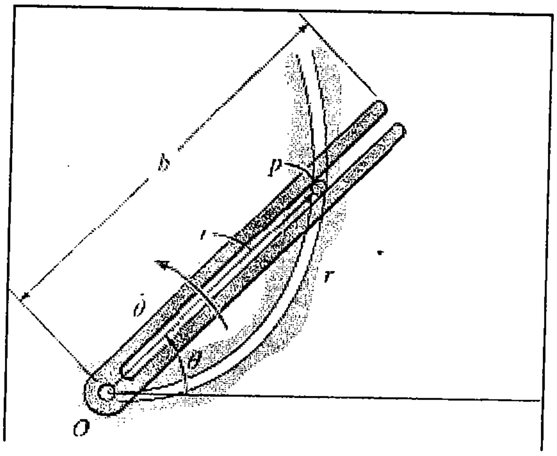
A truck is traveling along the horizontal circular curve of radius r=60m with speed of 20m/s which is increasing at the rate $3 m/s^2$. Determine the truck's radial and transverse components of acceleration.



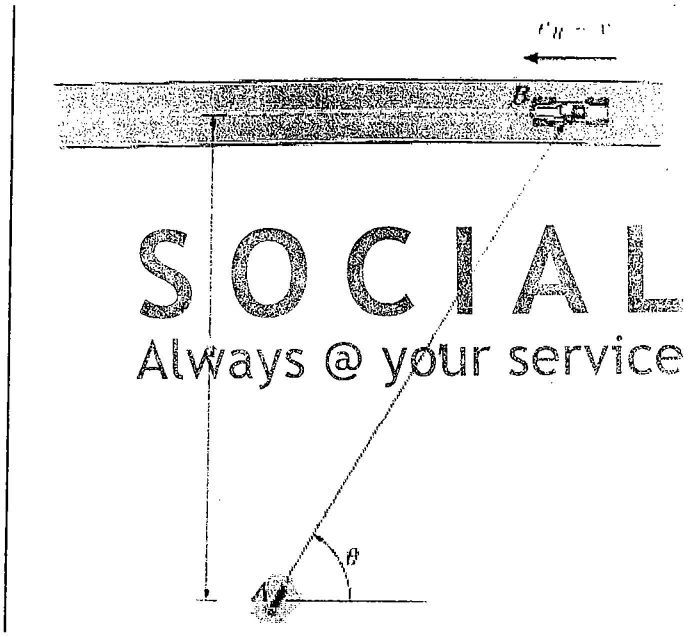
$a_r = -6.667 \, m/s^2$
$a_\theta = 3 \, m/s^2$

Problem 14

The slotted link is pinned at O, and as a result of the constant angular velocity $\theta' = 3 rad/s$ it drives the peg P for a short distance along the spiral guide $r = 0.4\theta$ where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



Problem 15
A cameraman standing at *A* is following the movement of a race car, *B*, which is traveling along a straight track at a constant speed 24m/s. Determine the angular rate at which he must turn in order to keep the camera directed on the car at the instant $\theta = 60°$. Given a=30m.



Homework 1

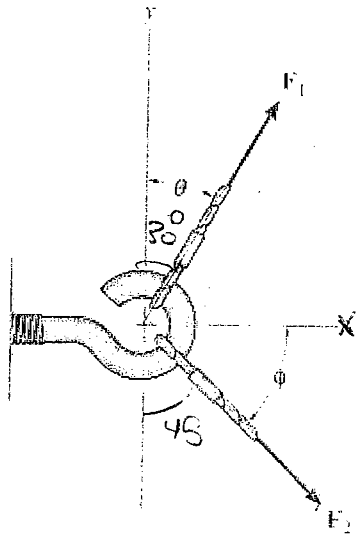
Problem 1
Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and its direction, measured counterclockwise from the positive x axis.
Given:

$F1 = 250$ N

$F2 = 375$ N

$\theta = 30°$

$\varphi = 45°$



Problem 2
Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and its direction measured counterclockwise from the positive u axis.
Given:

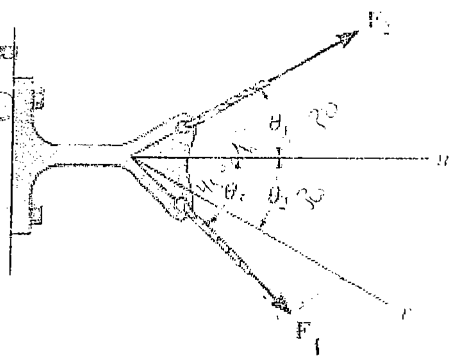
$F1 = 25$ N

$F2 = 50$ N

$\theta_1 = 30°$

$\theta_2 = 30°$

$\theta_3 = 45°$



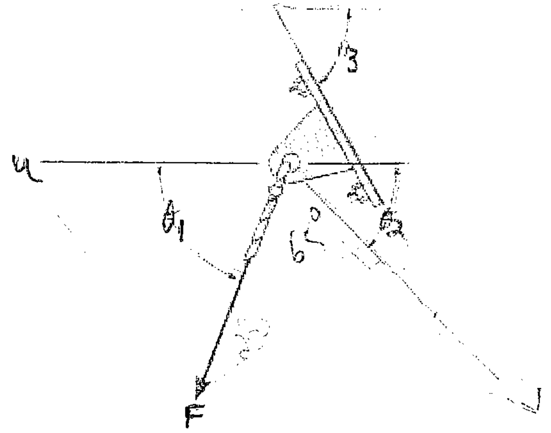
Problem 3
Determine the components of the $\mathbf{F}$ force acting along the u and v axes.
Given:

$\theta_1 = 70°$

$\theta_2 = 45°$

$\theta_3 = 60°$

$F = 250$ N



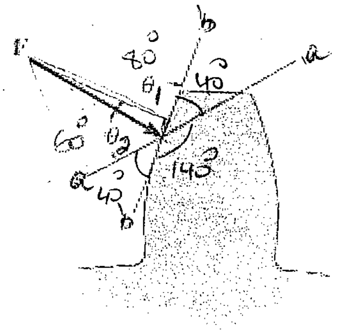
Problem 4

The component of force **F** acting along line *aa* is required to be *30 N*. Determine the magnitude of **F** and its component along line *bb*.

Given:

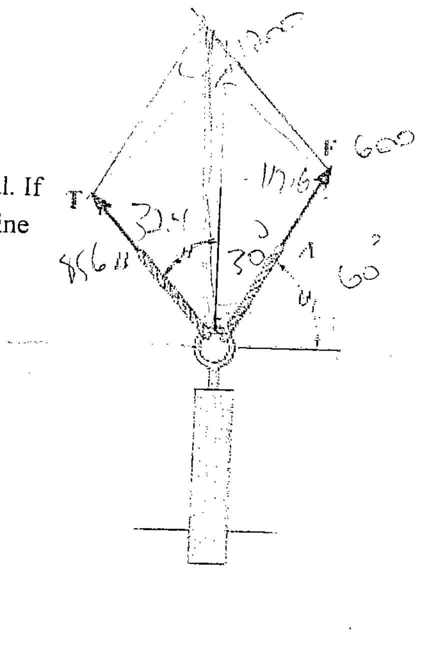
$\theta_1 = 80°$

$\theta_2 = 60°$



Problem 5

The post is to be pulled out of the ground using two ropes *A* and *B*. Rope *A* is subjected to force of *600 N* and is directed at angle $\theta_1 = 60°$ from the horizontal. If the resultant force acting on the post is to be *1200 N*, vertically upward, determine the force **T** in rope *B* and the corresponding angle θ.



Engineering Mechanics
Homework 1

① • $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

$= \sqrt{(F_1 \sin\theta + F_2 \cos\varphi)^2 + (F_1 \cos\theta - F_2 \sin\varphi)^2}$

$= \sqrt{(250 \sin 30 + 375 \cos 45)^2 + (250 \cos 45 - 375 \sin 45)}$

$\simeq 400N$

• $\frac{400}{\sin 15^\circ} = \frac{250}{\sin \alpha} \Rightarrow \sin\alpha = 0.6 \Rightarrow \alpha = 37.1^\circ$

$\Rightarrow \theta =$ (in course)

② $F_R^2 = f_1^2 + f_2^2 - 2f_1f_2 \cos(180 - 75^\circ)$

$= 25^2 + 50^2 - 2(25)(50)\cos(105^\circ)$

$= 3772 N^2$

$\Rightarrow F_R = 61.4N$

• $\frac{61.4}{\sin 105} = \frac{50}{\sin\alpha} \Rightarrow \alpha = 51.9^\circ$

$\Rightarrow \beta = 51.9^\circ - 45^\circ = 6.9^\circ$

(β being angle btw $\vec{F_R}$ and u-axis measured counterclockwise from +ve u-axis).

③ $F = F_x + F_y \Rightarrow \vec{F} = F_x\vec{u} + F_y\vec{v}$

$= F\cos 70^\circ \vec{u}$

(4) component of F acting on aa $= F\cos 60^\circ = 30$

$\Rightarrow F = \dfrac{30}{\cos 60^\circ} = 60N$

$\rightarrow$ mag of $F = 60N$

- component of F along bb $= 60\cos 80^\circ = 10.4N$

(5) $\vec{F_R} = \vec{F} + \vec{T}$

$= (600\cos 60^\circ \vec{i} + 600\sin 60^\circ \vec{j}) + (-T\sin\theta \vec{i} + T\cos\theta \vec{j})$

$= (600\cos 60^\circ - T\sin\theta)\vec{i} + (600\sin 60^\circ + T\cos\theta)\vec{j}$

F_R is vertically upwards $\Rightarrow$ no x component

$\Rightarrow 600\cos 60^\circ = T\sin\theta$ ___ (1)

$\Rightarrow F_R^2 = (600\sin 60^\circ + T\cos\theta)^2$

$1200^2 = (520 + T\cos\theta)^2$

square root both sides

$\pm 1200 = 520 + T\cos\theta$

1. $T\cos\theta = 1720$

$\Rightarrow \tan\theta = \dfrac{520}{1720} \Rightarrow \theta = 16.8^\circ$ (rej)

2. $T\cos\theta = 680$ ___ (2)

$\Rightarrow \tan\theta = \dfrac{520}{680} \Rightarrow \theta = 37.4^\circ$

$\Rightarrow 520 = T\sin(37.4)^\circ \Rightarrow T = 856\ N$

HW

Homework 2

Problem 1
Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.
Given:

$F1$ = 850 N

$F2$ = 625 N

$F3$ = 750 N

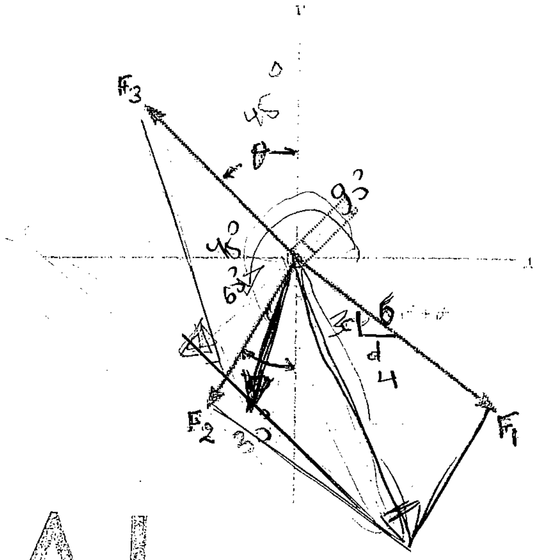
θ = 45°

φ = 30°

c = 3

d = 4

ANS:
$F_R = 546$ N
$\beta = 252.6°$



IMP

Problem 2
Three forces act on the bracket. Determine the magnitude and direction θ of **F**1 so that the resultant force is directed along the positive x' axis and has a magnitude of **1KN**.
Given:

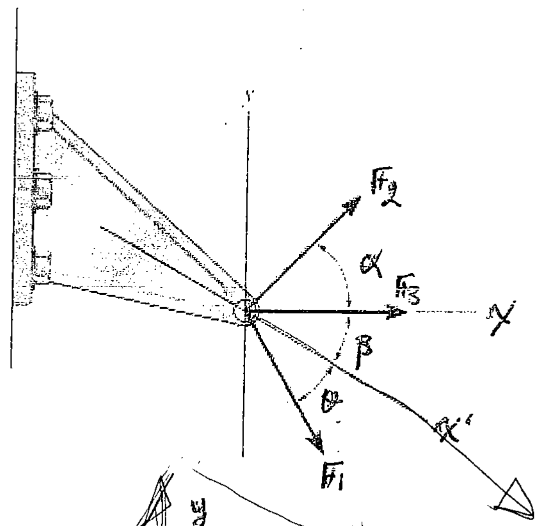
$F2$ = 450 N

$F3$ = 200 N

α = 45°

β = 30°

ANS:
$F_1 = 889$ N
$\theta = 37°$



Problem 3
Determine the magnitude and direction, measured counterclockwise from the positive x' axis, of the resultant force of the three forces acting on the bracket.
Given:

$F1$ = 300 N

$F2$ = 200 N

F_3 = 180 N

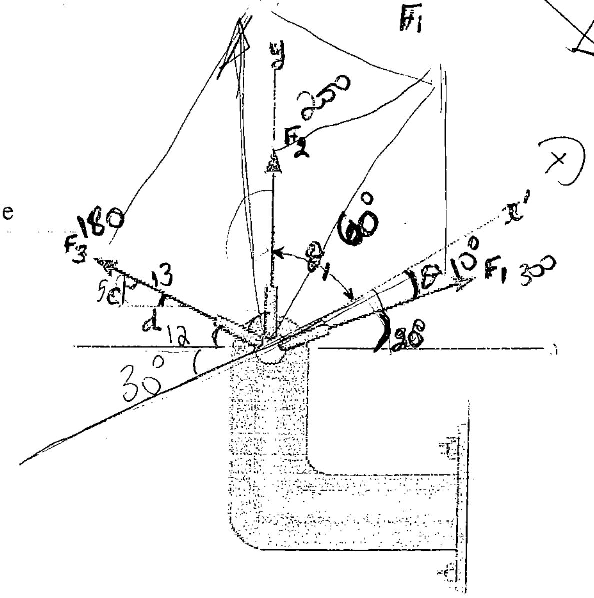
$\theta = 10°$

$\theta_1 = 60°$

$c = 5$

$d = 12$

ANS:
$F_R = 389$ N
$\phi = 72.7°$ $\psi = 42.7°$



Homework 2

Problem 1:

$F_{Rx} = \frac{4}{5}F_1 - F_3\sin45^\circ - F_2\sin30^\circ$

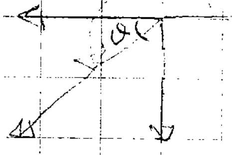
$= \frac{4}{5}(850) - 750\sin45^\circ - 625\sin30^\circ$

$= -162.83 = 162.83 \leftarrow$

$F_{Ry} = F_3\cos45^\circ - \frac{3}{5}F_1 - F_2\cos30^\circ$

$= 750\cos45^\circ - \frac{3}{5}(850) - 625\cos30^\circ$

$= -520.94 = 520.94 \downarrow$



$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2}$

$= 546N$

$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1}3.199 = 72.6^\circ$

$\Rightarrow \alpha = 180 + 72.6 = 252.6^\circ$

(measure counterclockwise from the x axis)

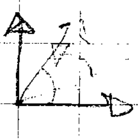
Problem 2:

115.25 →

$F_{Rx} = 300\cos30^\circ - 180\left(\frac{12}{13}\right) = \cancel{\ldots}$

$F_{Ry} = 200 + 300\cos70^\circ + 180\left(\frac{5}{13}\right)$

$= 371.84N$



$\Rightarrow \theta = \tan^{-1} = 72.7^\circ$

$\Rightarrow \varphi = 72.7^\circ - 30^\circ = 42.7^\circ$

$F_R = 389N.$

HOMEWORK 2

Problem 13

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(-F_3 \sin 45^\circ - F_2 \sin 30^\circ + F_1 \frac{4}{5}\right)^2 + \left(F_3 \cos 45^\circ - F_2 \cos 30^\circ - F_1 \frac{3}{5}\right)^2}$$

$$= \sqrt{\left(-750 \sin 45^\circ - 625 \sin 30^\circ + \frac{850 \times 4}{5}\right)^2 + \left(750 \cos 45 - 625 \cos 30^\circ - \frac{850 \times 3}{5}\right)^2}$$

$$= 545.7 \simeq 546 \text{ N}$$

$$\cos\alpha = \frac{|F_y|}{|F|} \Rightarrow \alpha = 162.5$$

$$\beta = +90 = 252.6^\circ$$

$$\cancel{\beta = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{-520.94}{-162.83} =}$$

$$\vec{F_R} = F_x \vec{i} + F_y \vec{j} + F_3 x'$$

$$= \left(F_2 \cos\alpha + F_3 + F_1 \cos(\theta+\beta)\right)\vec{i} + \left(F_2 \sin\alpha - F_1 \sin(\theta+\beta)\right)\vec{j}$$

$$+ \left(F_2 \cos(\alpha+\beta) + F_3 \cos\beta + F_1 \cos\theta\right) x'$$

- $450 \cos 45^\circ + 200 + F_1 \cos(\theta + 30) = 0$
- $450 \sin 45^\circ - F_1 \sin(\theta + 30^\circ) = 0$

$$F_1 \cos(\theta + 30^\circ) = -450 \cos 45 - 200$$

$$F_1 \sin(\theta + 30^\circ) = 450 \sin 45^\circ$$

$$F_1^2 \cos^2(\theta + 30^\circ) = 450^2 \cos^2 45 + 200^2 + 400 \cdot 450 \cos 45$$

$$F_1^2 \sin^2(\theta + 30^\circ) = 450^2 \cos^2 45$$

$$\cancel{F_1^2 =}$$

$$F_R = \sqrt{F_{x'}^2 + F_y^2}$$

$$= \sqrt{\left(F_1 \cos\theta + F_2 \cos 60^\circ + F_3 \frac{12}{13}\right)^2 + \left(F_2 + F_1 \cos 70^\circ + F_3\right.}$$

Problem 2

$F_R = F_x \vec{i} + F_y \vec{j}$

$(1000\cos\beta)\vec{i} - (1000\sin\beta)\vec{j} = (F_3 + F_2\cos\alpha + F_1\cos(\theta+\beta))\vec{i} + (F_2\sin\alpha - F_1\sin(\theta+\beta)$

$1000\cos 30^\circ = 200 + 450\cos 45^\circ + F_1\cos(\theta + 30^\circ)$

$F_1\cos(\theta + 30^\circ) = 347.83$

$-1000\sin 30^\circ = 450\sin 45^\circ - F_1\sin(\theta + 30^\circ)$

$F_1\sin(\theta + 30^\circ) = 818.20$

$F_1^2 = 790{,}436.95$

$\Rightarrow F_1 = 889\ N$

replace : $889\cos(\theta + 30^\circ) = 347.83$

$\cos(\theta + 30^\circ) = 0.39$

$\cos\theta\cos 30^\circ - \sin\theta\sin 30^\circ = 0.39$

$0.866\cos\theta - 0.5\sin\theta = 0.39$ —— (1)

$\sin(\theta + 30^\circ) = 0.92$

$\sin\theta\cos 30^\circ + \cos\theta\sin 30^\circ = 0.92$

$0.5\cos\theta + 0.866\sin\theta = 0.92$ —— (2)

$\Rightarrow \cos\theta = 0.8 \Rightarrow \theta = 36.8^\circ$

$\sin\theta = 0.6 \Rightarrow \theta = 36.8^\circ$

$F_R = \sqrt{\cancel{F_x}\left(F_1\cos 20^\circ - F_3\frac{12}{15}\right)^2 + \left(200 + F_1\cos 70^\circ + F_3\frac{5}{13}\right)^2}$

$= \left(300\cos 20^\circ - \frac{180\times 12}{15}\right)^2 + \left(200 + 300\cos 70^\circ + \frac{180\times 5}{13}\right)^2$

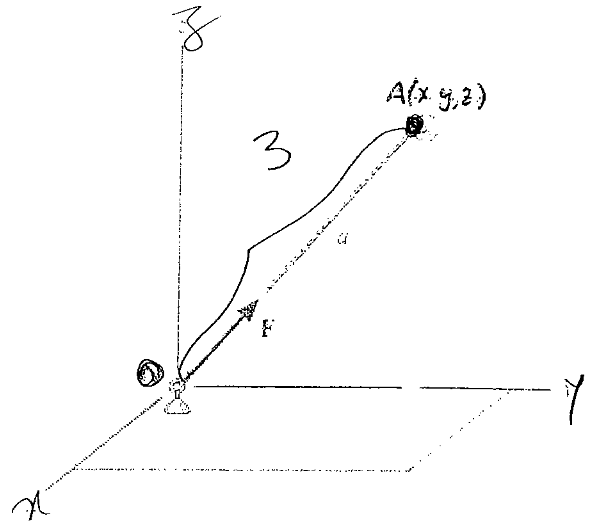
$= 396.6$

$\frac{371.84}{137.9}$

Homework 3

Problem 1
The cable *OA* exerts force $\mathbf{F} = \{40\mathbf{i} + 60\mathbf{j} + 70\mathbf{k}\}$ N on point *O*. If the length of the cable is $L = 3m$, what are the coordinates (x, y, z) of point *A*?

ANS: (1.2 , 1.8 , 2.1)



Problem 2
Determine the position $(x, y, 0)$ for fixing cable *BA* so that the resultant of the forces exerted on the pole is directed along its axis, from *B* toward *O*, and has magnitude of 1 kN. Also, what is the magnitude of force F3?
Given:

$F1$ = 500 N

$F2$ = 400 N

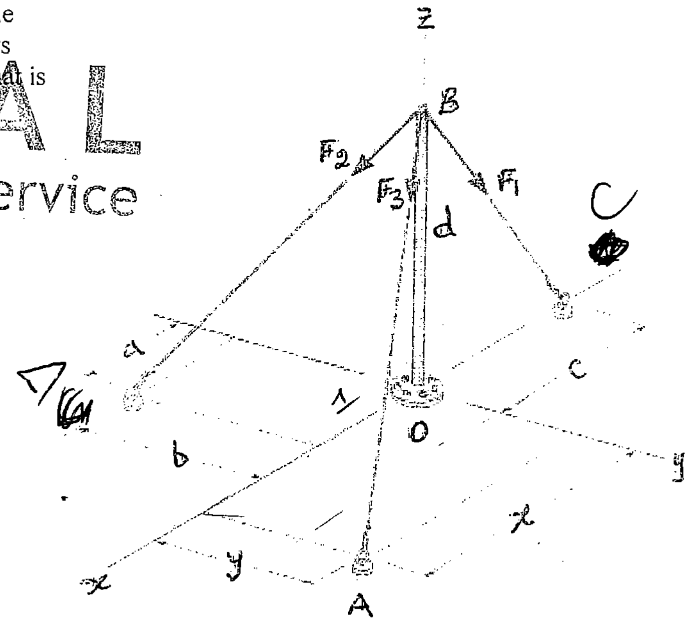
a = 1 m

b = 2 m

c = 2 m

d = 3 m

ANS: $F_3 = 380$ N



$dA = 4\pi r^2$

$F = qE = ma = mg$

$mg = qE$

$E = \frac{q}{A} = \frac{Q_{in}}{2\varepsilon_0 A}$ σ

$\Rightarrow$

$\Phi_{disk} = \int E\,dA = E \int \cancel{2\pi R\,dA} = \frac{1}{4}\frac{Q}{\varepsilon_0}$

$E\,4\pi R^2 = \frac{1}{4}\frac{Q}{\varepsilon_0}$

$4\pi R^2 = \frac{1}{4}\frac{Q}{\varepsilon_0}$

$\frac{1QR^2}{4\pi\varepsilon_0 b^2} = \frac{1}{4}\frac{Q}{\varepsilon_0}$ $\qquad \frac{R^2}{(R+b)^2} = \frac{1}{4}$

$\frac{R^2}{(R+B)^2} = \frac{1}{4}$ $\qquad 4R^2 = R^2 + b^2 - 2RB$

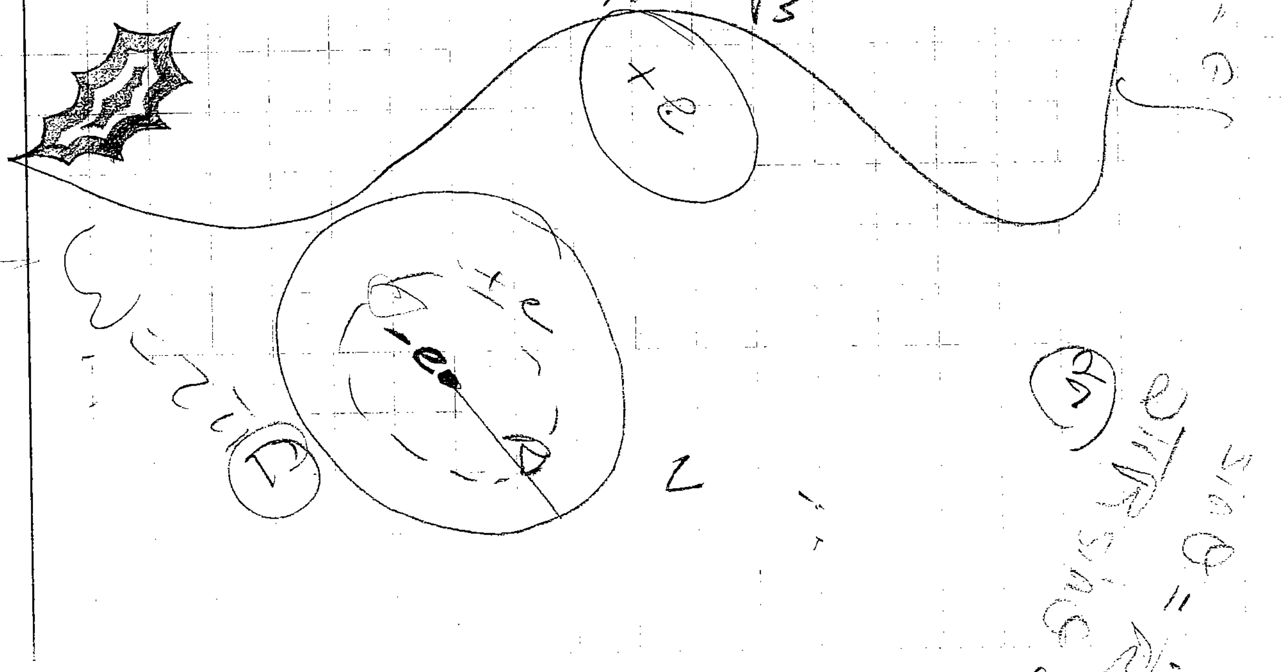
$3R^2 = b^2 - 2Rb$

$4R^2 = R^2 + B^2$

$3R^2 = B^2$

$R^2 = \frac{B}{\sqrt{3}}$

$\Phi = \int E\,dA$



Homework 3

$\vec{F} = 40\vec{i} + 60\vec{j} + 70\vec{k}$

$u_{OA} \cdot |\vec{F}| = \vec{F}$

$|\vec{F}| = 100.5$

$\Rightarrow 100.5\, x_A + 100.5\, y_A + 100.5\, z_A = \cancel{40\vec{i} + 60\vec{j} + 70\vec{k}}$

$100.5\, x_A = 40$

$\Rightarrow x_{u_{OA}} = 2.8$

$y_{u_{OA}} = 1.675$

$z_{OA} = 1.44$

$u_{OA} = \frac{|\vec{OA}|\, \vec{OA}}{|\vec{OA}|} = \vec{OA} =$

$|\vec{OA}| = 3$ $\quad \vec{F} = 40\vec{i} + 60\vec{j} + 70\vec{k}$

$|\vec{F}| = 100.5$

$|\vec{F}|\, \vec{u}_{OA} = \vec{F}$

$\vec{u}_{OA} = \frac{\vec{F}}{|\vec{F}|} = \frac{40}{100.5}\vec{i} + \frac{60}{100.5}\vec{j} + \frac{70}{100.5}\vec{k}$

$= 0.4\vec{i} + 0.6\vec{j} + 0.7\vec{k}$

$\Rightarrow \vec{u}_{OA} = \frac{\vec{OA}}{|\vec{OA}|} \Rightarrow \vec{OA} = 3(0.4\vec{i} + 0.6\vec{j} + 0.7\vec{k})$

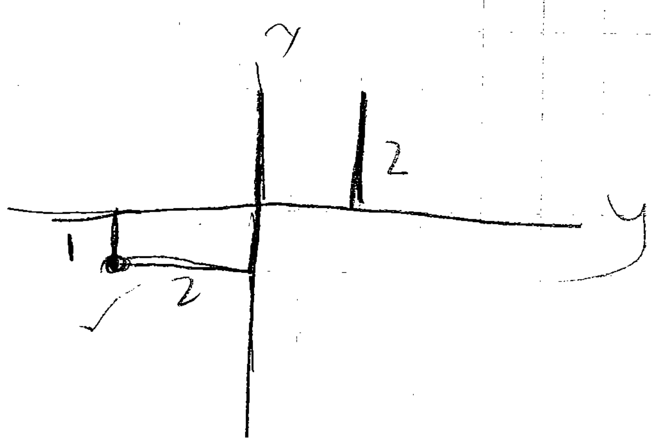
$= 1.2\vec{i} + 1.6\vec{j} + 2.1\vec{k}$

$F_R = 1KN = 1000$

$O(0,0,0)$

$B(0,0,3)$

$\vec{OB}\ (0,0,3)\ ;\ |\vec{OB}| = 3$

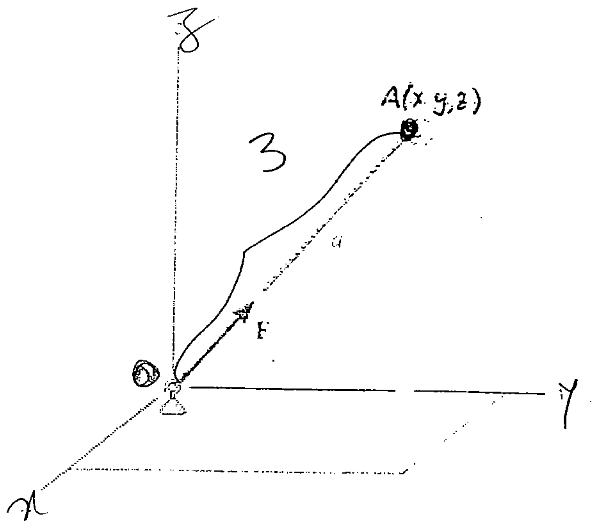


Homework 3

Problem 1

The cable *OA* exerts force $\mathbf{F} = \{40\mathbf{i} + 60\mathbf{j} + 70\mathbf{k}\}$ N on point *O*. If the length of the cable is $L = 3m$, what are the coordinates (x, y, z) of point *A*?

ANS: (1.2 , 1.8 , 2.1)



Problem 2

Determine the position $(x, y, 0)$ for fixing cable *BA* so that the resultant of the forces exerted on the pole is directed along its axis, from *B* toward *O*, and has magnitude of 1 kN. Also, what is the magnitude of force F_3?

Given:

$F1$ = 500 N

$F2$ = 400 N

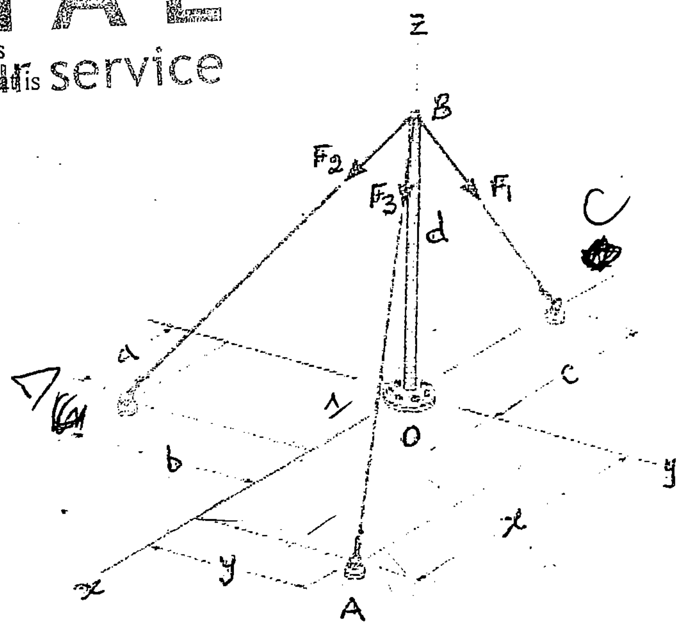
a = 1 m

b = 2 m

c = 2 m

d = 3 m

ANS: $F_3 = 380$ N



Homework 3

$\vec{F} = 40\vec{i} + 60\vec{j} + 70\vec{k}$

$\vec{u}_{OA} \cdot |\vec{F}| = \vec{F}$

$|\vec{F}| = 100.5$

$\Rightarrow 100.5\,x_A + 100.5\,y_A + 100.5\,z_A = 40\vec{i} + 60\vec{j} + 70\vec{k}$

$100.5\,x_A = 40$

$\Rightarrow x_{u_{OA}} = 2.8$

$y_{u_{OA}} = 1.675$

$z_{u_{OA}} = 1.44$

$u_{\vec{OA}} = \dfrac{|\vec{OA}|\vec{OA}}{|\vec{OA}|} = \vec{OA} =$

$|\vec{OA}| = 3$ $\qquad \vec{F} = 40\vec{i} + 60\vec{j} + 70\vec{k}$

$|\vec{F}| = 100.5$

$|\vec{F}|\,\vec{u}_{OA} = \{\vec{F}\}$

$\vec{u}_{OA} = \dfrac{\{\vec{F}\}}{|\vec{F}|} = \dfrac{40}{100.5}\vec{i} + \dfrac{60}{100.5}\vec{j} + \dfrac{70}{100.5}\vec{k}$

$= 0.4\vec{i} + 0.6\vec{j} + 0.7\vec{k}$

$\Rightarrow \vec{u}_{OA} = \dfrac{\vec{OA}}{|\vec{OA}|} \Rightarrow \vec{OA} = 3(0.4\vec{i} + 0.6\vec{j} + 0.7\vec{k})$

$= 1.2\vec{i} + 1.6\vec{j} + 2.1\vec{k}$

$F_R = 1\,kN = 1000$

$O(0,0,0)$

$B(0,0,3)$

$\vec{OB}\ (0,0,3)\ ;\ |\vec{OB}| = 3$

$dA = 4\pi r^2$

$F = qE = ma = mg$

$mg = qE$

$E = \frac{\sigma}{A} = \frac{Q_{in}}{2\varepsilon_0 A}$

$\Rightarrow$

$\Phi_{disk} = \int E\,dA = \frac{Q}{\varepsilon_0}$ $E \int 2\pi R\,dA$ $= \frac{1}{4}\frac{Q}{\varepsilon_0}$

$E\,4\pi R^2 = \frac{1}{4}\frac{Q}{\varepsilon_0}$

$\frac{kQ}{b^2}\,4\pi R^2 = \frac{1}{4}\frac{Q}{\varepsilon_0}$

$\frac{1QR^2}{4\pi\varepsilon_0 b^2} = \frac{1}{4}\frac{Q}{\varepsilon_0}$

$\frac{R^2}{(R+b)^2} = \frac{1}{4}$

$\frac{R^2}{(R^2+B^2)} = \frac{1}{4}$

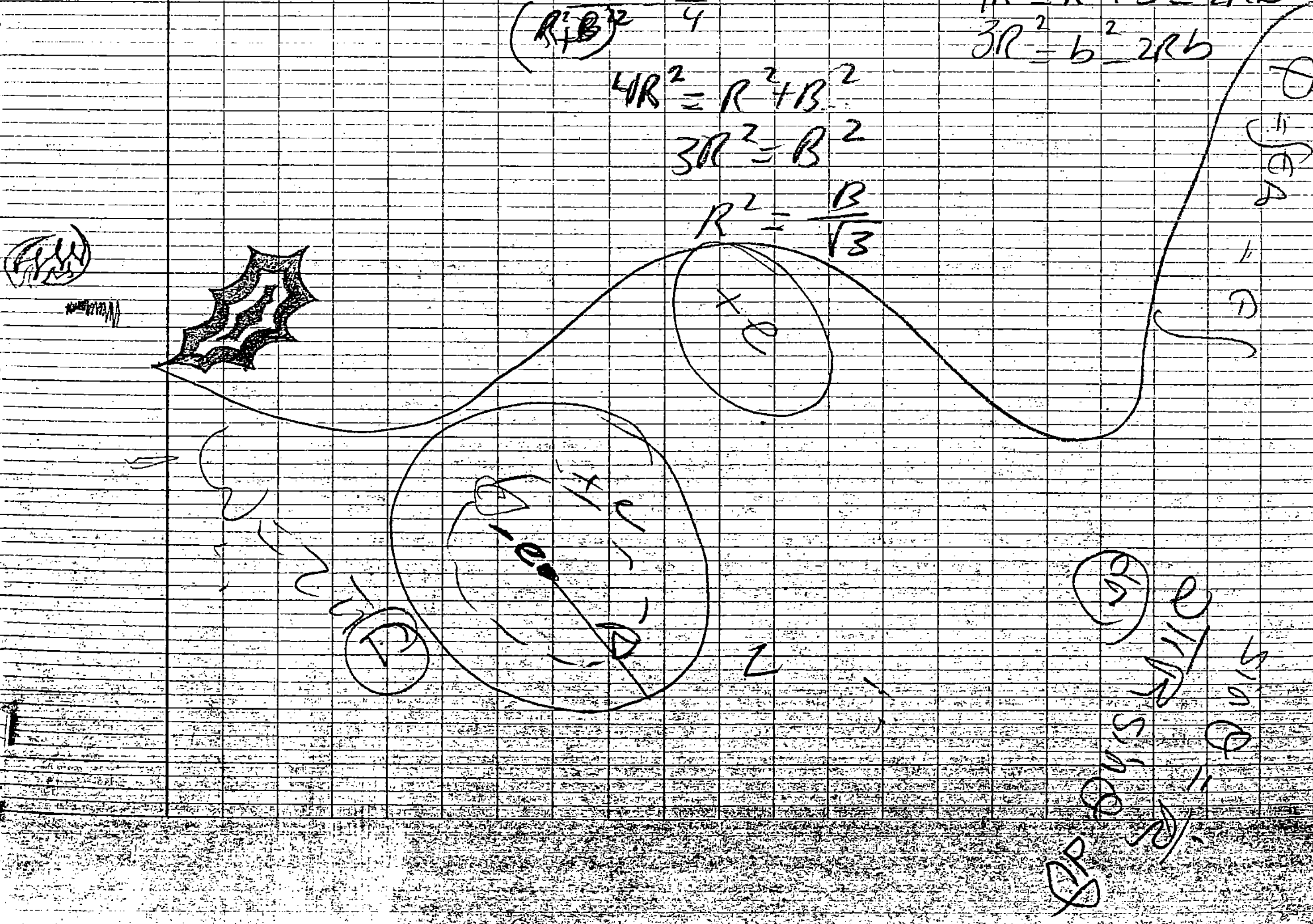
$4R^2 = R^2 + b^2 - 2RB$

$3R^2 = b^2 - 2Rb$

$4R^2 = R^2 + B^2$

$3R^2 = B^2$

$R^2 = \frac{B}{\sqrt{3}}$



Homework 4 (done)

Problem 1 (class)

Determine the maximum weight of the engine that can be supported without exceeding a tension of T_1 in chain *AB* and T_2 in chain *AC*.

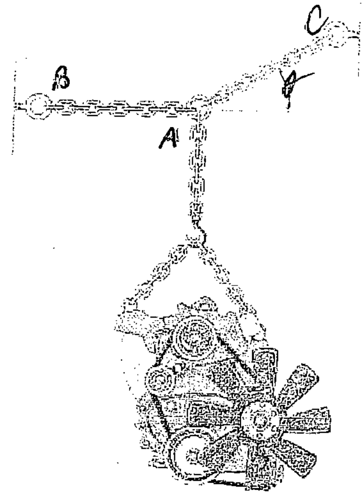
Given:

$\theta = 30°$

$T_1 =$ ~~4.5 kN~~ 450 N

$T_2 =$ ~~4.8 kN~~ 480 N

ANS: W = 240 N



Problem 2

The unstretched length of spring *AB* is δ ($L_0(AB)$) = 2m. If the block is held in the equilibrium position shown, determine the mass of the block at *D*.

Given:

$a = 3$ m

$b = 3$ m

$c = 4$ m

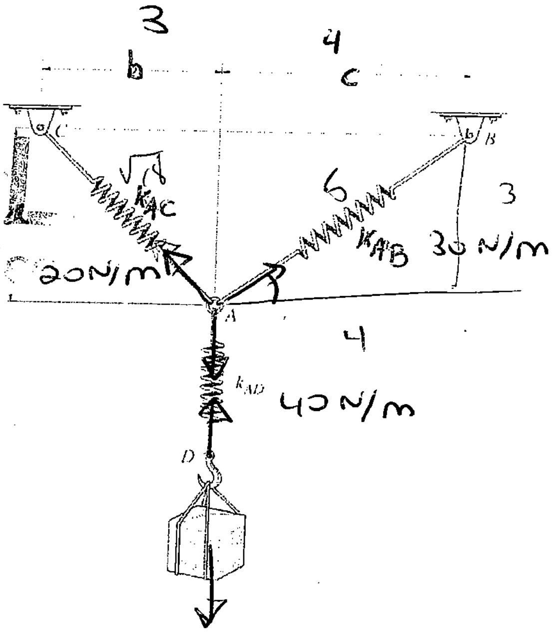
$k_{AB} = 30$ N/m

$k_{AC} = 20$ N/m

$k_{AD} = 40$ N/m

ANS.

$m_D = 12.8$ kg



Problem 3

Determine the force in each cable and the force **F** needed to hold the lamp of mass *M* in the position shown. *Hint:* First analyze the equilibrium at *B*; then, using the result for the force in *BC*, analyze the equilibrium at *C*.

Given:

$M = 4$ kg

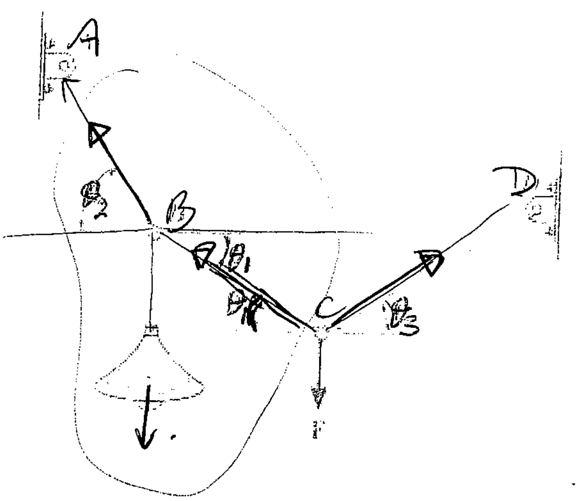
$\theta_1 = 30°$

$\theta_2 = 60°$

ANS:

$T_{CD} = 39.24$ N

$F = 39.24$ N



$\theta_3 = 30°$

Problem 4 (class)

The 30-kg block is supported by two springs having the stiffness shown. Determine the unstretched length of each spring.

Given:

$M = 30$ kg

$l_1 = 0.6$ m

$l_2 = 0.4$ m

$l_3 = 0.5$ m

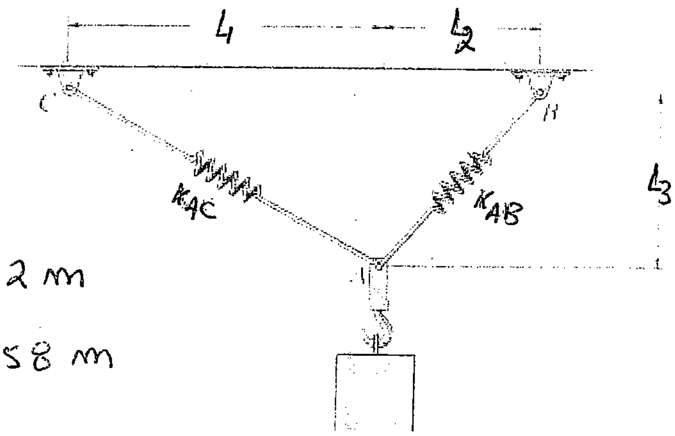
$k_{AC} = 1.5$ kN/m

$k_{AB} = 1.2$ kN/m

ANS:

$L_{AB} = 0.452$ m

$L_{AC} = 0.658$ m



Problem 5 ⇒ equilibrium

A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown.

Given:

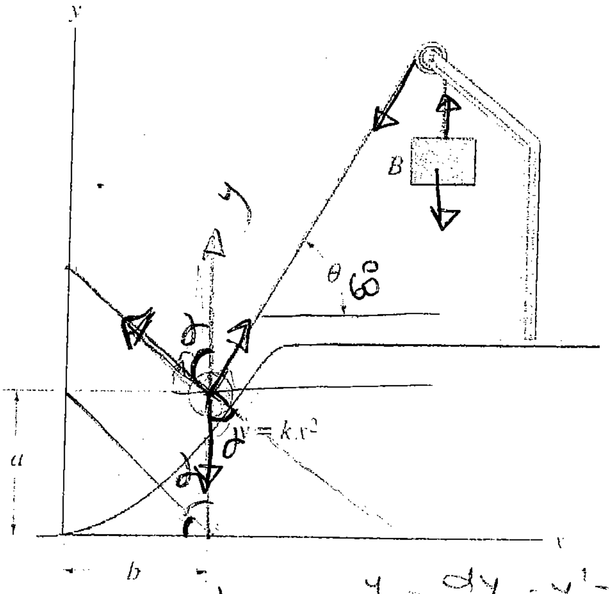
$a = 0.4$ m

$b = 0.4$ m

$\theta = 60°$

ANS. $F_N = 19.66$ N

$m_B = 3.58$ kg



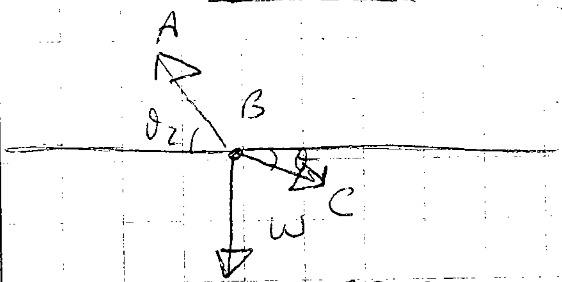
$\tan\alpha = \frac{y}{x} = \frac{dy}{dx} \to y' = 2kx$

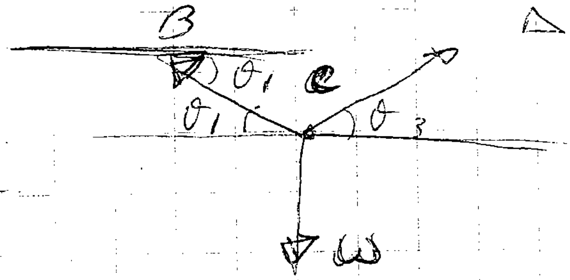
$y = kx^2 \to k = \frac{a}{b^2}$

$\therefore \text{slope} = \frac{2ab^2}{b^2} = \frac{2a}{b} = 2$

$\alpha = \tan^{-1} 2 = 63.4°$

HW 4 Problem 3





at B $\quad W = 39.2N$

$$\overset{+}{\rightarrow} \Sigma F_x = T_{BC}\cos\theta_1 - T_{BA}\cos\theta_2 = 0$$
$$\overset{+}{\uparrow} \Sigma F_y = -W - T_{BC}\sin\theta_1 + T_{BA}\sin\theta_2 = 0$$

$0.866 T_{BC} - 0.5 T_{BA} = 0$ $\qquad T_{BC} = 39.2$

$-0.5 T_{BC} + 0.866 T_{BA} = 39.2$ $\qquad T_{BA} = 67.9$

$\Rightarrow$

at C

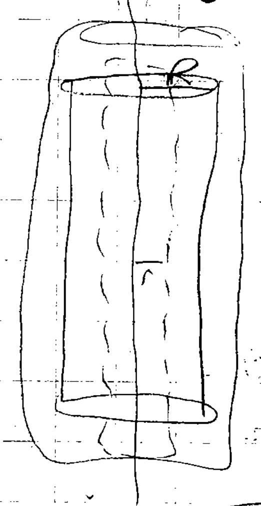
$$\overset{+}{\rightarrow} \Sigma F_x = T_{CD}\cos\theta_3 - T_{CB}\cos\theta_1 = 0$$
$$\overset{+}{\uparrow} \Sigma F_y = -F + T_{CD}\sin\theta_3 + T_{CB}\sin\theta_1 = 0$$

$$T_{CD} = \frac{T_{CB}\cos\theta_1}{\cos\theta_3} = 39.2$$

$$\Rightarrow F = 39.2\sin 30^\circ + 39.2\sin 30^\circ$$
$$= 39.2N$$

$\cos\theta = \frac{b}{b^2 - R^2}$

$E\int dA = \frac{Q}{4\varepsilon_0}$

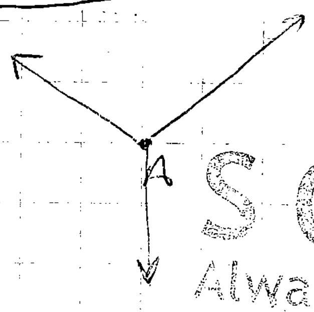


$dA = 2\pi R\, ds$
$= b\, d\theta$

$\phi = \int E\, dA = \frac{q_{in}}{\varepsilon_0} \quad q_{in} = \int \rho$

$E\int dA = \frac{1}{\varepsilon_0}\int \rho_0 \left(a - \frac{r}{b}\right) 2\pi r \ell\, dr$

Problem 2 HW4



$\Sigma F_x = 0 \Rightarrow T_{AB}\frac{4}{5} - T_{AC}\frac{3}{\sqrt{18}} = 0$

$\Sigma F_y = 0 - T_{AD} + T_{AB}\frac{3}{5} + T_{AC}\frac{3}{\sqrt{18}} = 0$

$T_{AB} = k\Delta\ell$
$= 30(5-2) = 90N$
$\Rightarrow 90 \times \frac{4}{5} \cdot \frac{\sqrt{18}}{3} = T_{AC} = 101.8$

$54 + \cancel{101.8} = T_{AD} = \cancel{158.8}\ 126$

$\cancel{T_{AD} = W = mg}$

$T_{AD} = W = mg$
$\Rightarrow m = 12.8\ kg$

Homework 5

Problem 1

Determine the magnitude and directional sense of the resultant moment of the forces at *A* and *B* about point *P*.

Given:

$F_1 = 40$ kN

$F_2 = 60$ kN

$\theta_1 = 30°$

$\theta_2 = 45°$

$a = 5$ m

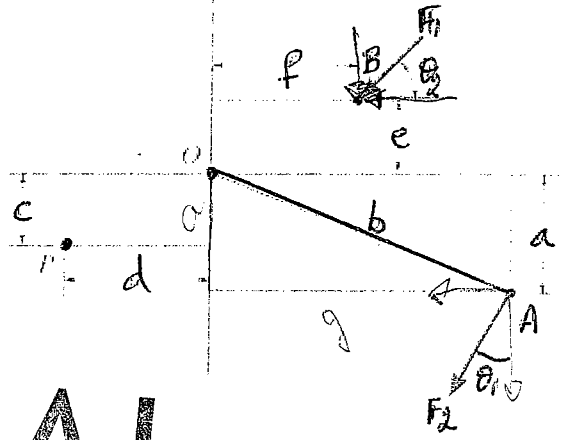
$b = 13$ m

$c = 3$ m

$e = 3$ m

$d = 6$ m

$f = 6$ m



Problem 2

Determine the angle θ (0 <= θ <= 90 deg) so that the force **F** = 100N develops a clockwise moment $M = 20\ N.m$ about point *O*.

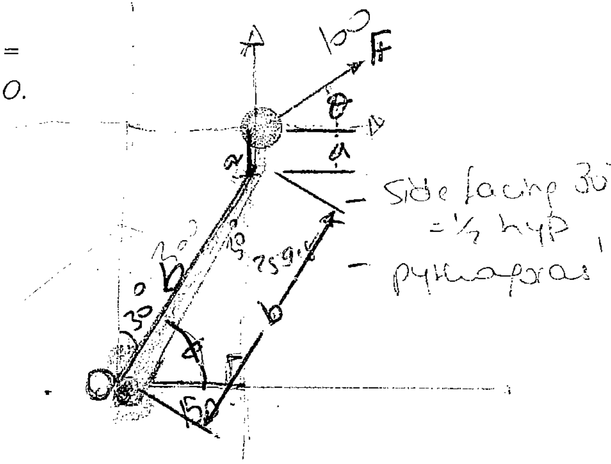
Given:

$\varphi = 60°$

$b = 300$ mm

$a = 50$ mm

ANS: $\theta = 28.6°$



Problem 3

Determine the magnitude and directional sense of the moment of the forces about point *P*.

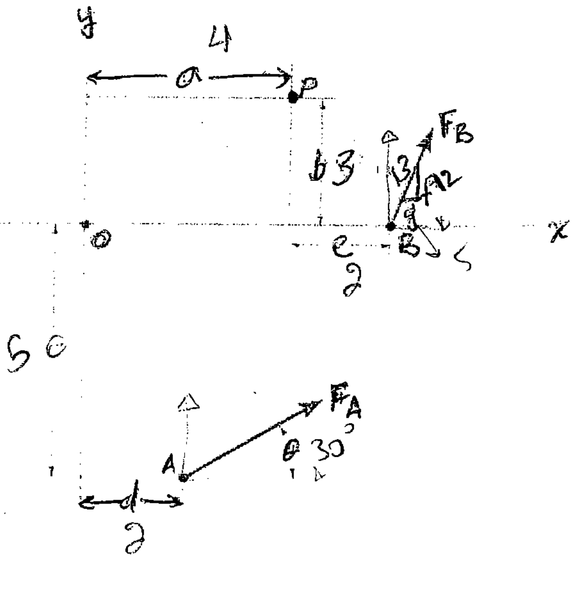
Given:

$FA = 400$ N

$FB = 260$ N

$\theta = 30°$

$a = 4$ m



* Max moment occurs when the force is ⊥ to the line between A & the point of application of the force.

$b = 3$ m

$c = 5$ m

$d = 2$ m

$e = 2$ m

$f = 12$

$g = 5$

ANS: $M_o = 3.57$ (positive C.C.W)

vector

Problem 4
A force **F** = 40N is applied to the wrench. Determine the moment of this force about point *O*. Solve the problem using both a scalar analysis and a vector analysis.
Given:

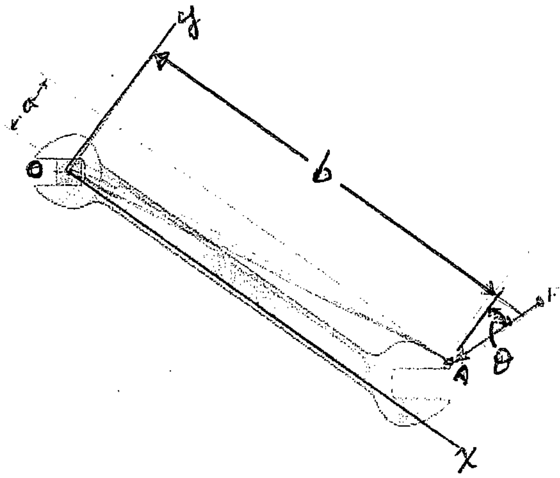
$F = 40$ N

$\theta = 20°$

$a = 30$ mm

$b = 200$ mm

ANS: $|M_o| = 7.11$ Nm



Problem 5
Determine the direction θ (0° ≤ θ ≤ 180°) of the force **F** = 200 N so that it produces **(a)** the maximum moment about point *A* and **(b)** the minimum moment about point *A*. Compute the moment in each case.
Given:

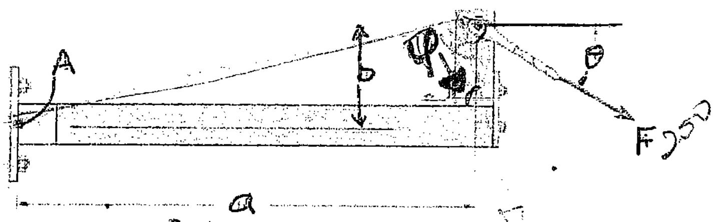
$a = 2.0$ m

$b = 0.5$ m

Max. moment occurs when the force is ⊥ to the line between A and the point of application of the force.

$M_{max} = F\sqrt{a^2+b^2}$

$\theta = 90 - \tan^{-1}\left(\frac{200}{30}\right)$



Problem 6
If the resultant moment about point *A* is *M* = *4800 N.m* clockwise, determine the magnitude of F_3 if

$F1 = 300$ N and $F2 = 400$ N.

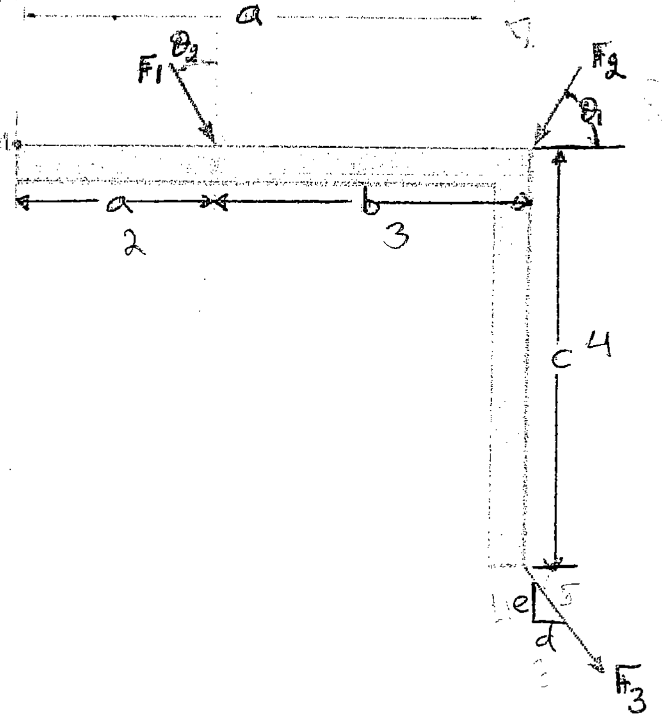
Given:

$\theta_1 = 60°$ $a = 2$ m $C = 4$ m

$\theta_2 = 30°$ $b = 3$ m $d = 3$

$e = 4$

ANS: $F_3 = 1.593$ kN



70419194

Homework 5

Problem 1:

$\Sigma M = +40\cos 45^\circ (6) - 40\sin 45^\circ (12) + 60\sin 30^\circ (2)$

$- 60\cos 30^\circ (\cancel{2})^{2+6}$ $\quad \cdot g = \sqrt{b^2 + a^2} = 12$

$= -853.24\ kNm$

Problem 2:

$\Sigma M = -20\,Nm$

$\Rightarrow -20 = -(100\cos\theta \times 309.8)\times 10^{-3} + (100\sin\theta \times 150)\times 10^{-3}$

$-20 = -30980\cos\theta + 15000\sin\theta$ — (1)

with: $100\ \cancel{\ } = 100\cos\theta + 100\sin\theta$ — (2)

$\cos\theta = 0.76$ $\quad \sin\theta = 0.24$

$\Rightarrow \theta =$

Problem 3:

$\Sigma M = +(280\cdot\frac{5}{13}\times 2 + 280\cdot\frac{12}{13}) + (400\cos 30^\circ \times 8)$

$-(400\sin 30^\circ \times 2)$

$= 3151.26\ N = 3.15\ kN$

Problem 4:

Scalars $\quad \Sigma M = -(40\cos 20^\circ \times 200\times 10^{-3}) + (40\sin 20^\circ\, 30\times 10^{-3})$

$= -7.11\ Nm$

$\Rightarrow |M_O| = 7.11\ Nm$

Vectors $\quad \vec{M} = \vec{r}\times\vec{F}$

$\vec{r} = \vec{OA} = 200\times 10^{-3}\vec{i} + 30\times 10^{-3}\vec{j}$

$0.2\vec{i} + 0.03\vec{j}$

$\vec{F} = -13.68\vec{i} - 37.59\vec{j}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.2 & 0.03 & 0 \\ -13.68 & -37.59 & 0 \end{vmatrix} = \cancel{\vec{i}(\ ) - \vec{j}(\ )} + \vec{k}($

$= -7.11\vec{k}$

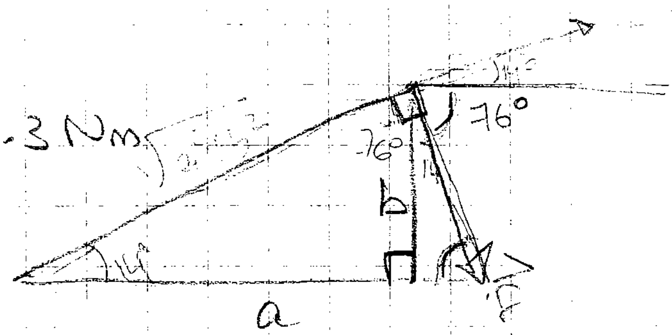
$\Rightarrow |\vec{M}| = |-7.11|\ Nm$

Problem 63

$M_{max} = F \times d$

$= 200 \times \sqrt{a^2+b^2} = 412.3 \text{ Nm}$

$\theta = 76°$



$M_{min} = 0$

Problem 65

$\Sigma M = -4800$

$-4800 = -(600\cos 30°) - (200\sin 60°) + (F_3 \cdot \frac{3}{5} \cdot 4) - (F_3 \cdot \frac{4}{5} \cdot 5)$

$-4800 = -2251.6 + 2.4F_3 - 4F_3$

$-2548.3 = -1.6F_3$

$\Rightarrow F_3 = 1593 \text{ kN} = 1.593 \text{ kN}$

$\Sigma M_O = -20$

$-20 = (-100\cos\theta \cdot 309.8\times10^{-3}) + F\sin\theta(150\times10^{-3})$

$-20 = -30.98\cos\theta + 15\sin\theta$

$30.98\cos\theta = 20 + 15\sin\theta$

$959.76\cos^2\theta = 400 + 225\sin^2\theta + 600\sin\theta$

$959.76(1-\sin^2\theta) = 400 + 225\sin^2\theta + 600\sin\theta$

$959.76 - 959.76\sin^2\theta - 400 - 225\sin^2\theta - 600\sin\theta = 0$

$-1184.76\sin^2\theta - 600\sin\theta + 559.76 = 0$

$\sin\theta = -0.98$

$\sin\theta = 0.45$

Homework 6

Problem 1
Replace the force at *A* by an equivalent force and couple moment at point *P*.
Given:

$F = 375$ N

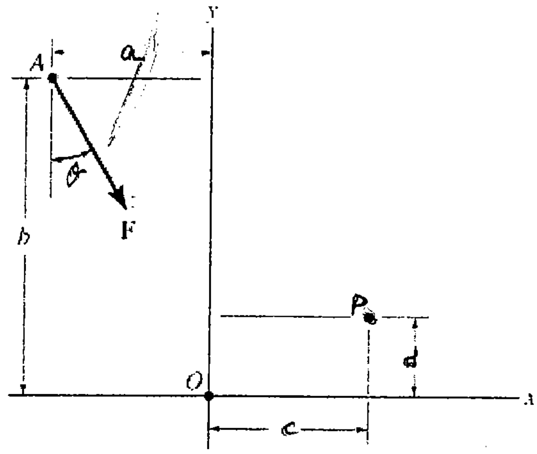
$a = 2$ m

$b = 4$ m

$c = 2$ m

$d = 1$ m

$\theta = 30$ °



Problem 2
Replace the force system by an equivalent resultant force and couple moment at point *P*.
Given:

$F1 = 60$ kN

$F2 = 85$ kN

$F3 = 25$ kN

$\theta = 45$ °

$a = 2$ m

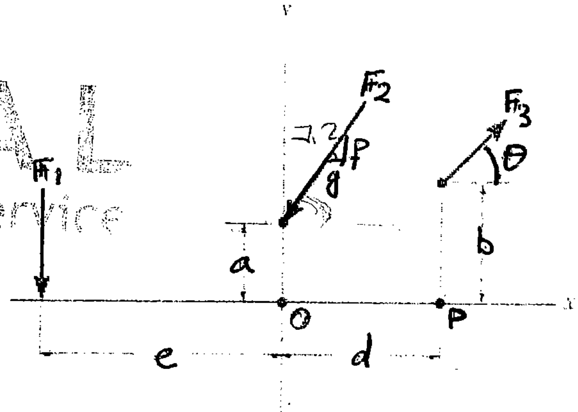
$b = 3$ m

$c = 6$ m

$d = 4$ m

$e = 3$

$f = 4$

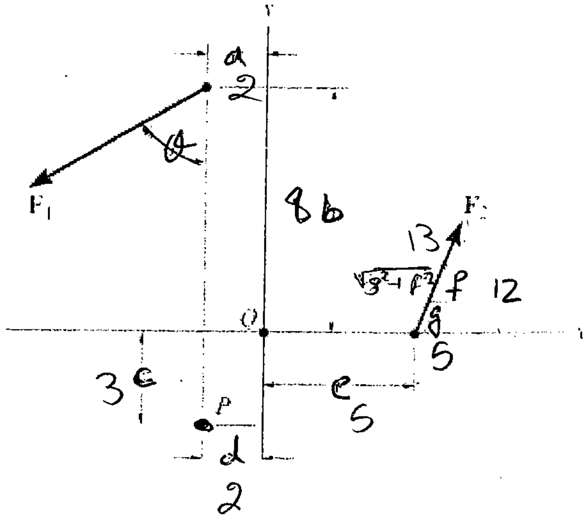


Problem 3
Replace the force system by an equivalent force and couple moment at point *P*.
Given:

$F1 = 430$ kN

$F2 = 260$ kN

$\theta = 60$ °



$a = 2$ m

$b = 8$ m

$c = 3$ m

$d = a$

$e = 5$ m

$f = 12$

$g = 5$

Problem 4

Replace the loading system acting on the post by an equivalent resultant force and couple moment at point *P*.

Given:

$F1 = 30$ kN

$F2 = 40$ kN

$F3 = 60$ kN

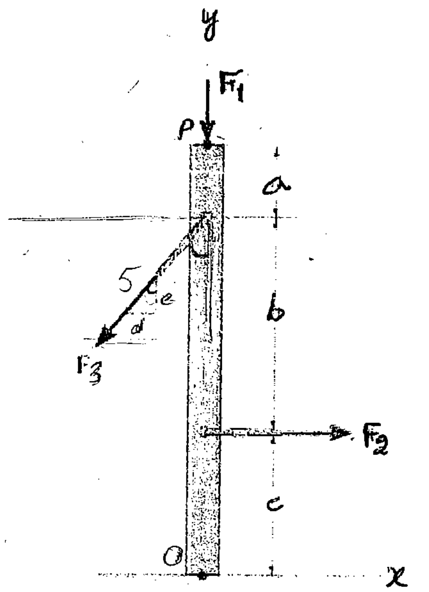
$a = 1$ m

$b = 3$ m

$c = 2$ m

$d = 3$

$e = 4$



? No reactions at pin & roller since, we don't want it to be in equilibrium...

Problem 5

Replace the loading on the frame by a single resultant force. Specify where the force acts , measured from end *A*.

where do we put resultant?

Given:

$F1 = 450$ N $\theta = 60°$

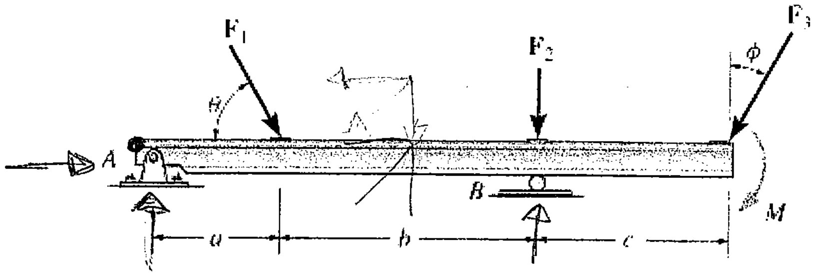
$F2 = 300$ N $M = 1500$ Nm

$F3 = 700$ N $\varphi = 30°$

$a = 2$ m

$b = 4$ m

$c = 3$ m



Engineering Mechanics (Homework 6)

① Proble 1

Force Summation:

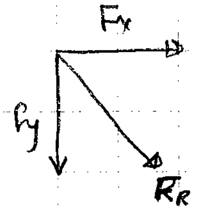
$\xrightarrow{+} \Sigma F_x = F\sin\theta = 375\sin 30^\circ = 187.5^\circ \rightarrow$

$+\uparrow \Sigma F_y = -F\cos\theta = -375\cos 30^\circ = 324.76 \downarrow$

$\Rightarrow F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

$= \sqrt{187.5^2 + 324.76^2}$

$= 375\ N$



Moment:

$+\circlearrowleft \Sigma M_P = -187.5(3) + 324.76(4)$

$= 736.54\ Nm$

② Proble 28

$\xrightarrow{+} \Sigma F_x = -85\left(\frac{9}{7.2}\right) + 25\cos 45^\circ = -88.57 = 88.57^\circ \leftarrow$

$+\uparrow \Sigma F_y = -60 - 85\left(\frac{4}{7.2}\right) + 25\sin 45^\circ = -88.9$

$= 88.9 \downarrow$

SOCIAL

Always @ your service

$\Rightarrow F_R = 125.4$

$\theta = \tan^{-1}\frac{F_y}{F_x} = 45.1^\circ$

$+\circlearrowleft \Sigma M_P = 60(7) + 85\left(\frac{4}{7.2}\right)(4) + 85\left(\frac{9}{7.2}\right)(2)$

$- 25\cos 45^\circ(3)$

$= 766\ kNm.$

③ Proble 3:

$\xrightarrow{+} \Sigma F_x = -430\sin 60^\circ + 260\left(\frac{5}{13}\right) = -272.4N = 272.4N \leftarrow$

$+\uparrow \Sigma F_y = -430\cos 60^\circ + 260\left(\frac{12}{13}\right) = 25N \uparrow$

$F_{Rx} = 273.54\ N$

$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = 5.24^\circ$

$\circlearrowleft \Sigma M_P = +430\sin 60^\circ(11) - 260\left(\frac{5}{13}\right)(3) + 260\left(\frac{12}{13}\right)(7)$

$= 5476.3\ kNm$

(4) Problem 43

$\xrightarrow{+} \Sigma F_x = F_2 - F_3\left(\frac{3}{5}\right) = 40 - 60\frac{3}{5} = 4\,kN \rightarrow$

$+\uparrow \Sigma F_y = -30 - 60\left(\frac{4}{5}\right) = -78\,kN = 78\,kN \downarrow$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{78^2 + 4^2} = 78.1\,kN$

$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = 87.06°$

$\circlearrowleft + \quad \Sigma M_P = +40(4) - 60\left(\frac{3}{5}\right)(1) = 124\,kNm \circlearrowleft$

(5) Problem 5

$\xrightarrow{+} \Sigma F_x = 450\cos 60° - 700\sin 30° = -125\,N = 125\,N \leftarrow$

$+\uparrow \Sigma F_y = -450\sin 60° - 300 - 700\cos 30° = -1295.9$

$F_R = 1301.9\,N \qquad = 1295.9 \downarrow$

$\theta = \tan^{-1}\frac{F_y}{F_x} = 84.5°$

$M_{RA} = \Sigma M_A$

$-1295.9(d) = -450\sin 60°(2) - 300(6) - 700\cos 30°(9)$

$\Rightarrow d = 6.2\,m$

Homework 7

?? why is it ⊥ ??
not diagonal

Problem 1

Determine the magnitude of the reactions on the beam at *A* and *B*. Neglect the thickness of the beam.

Given:

$F1 = 600$ N

$F2 = 400$ N

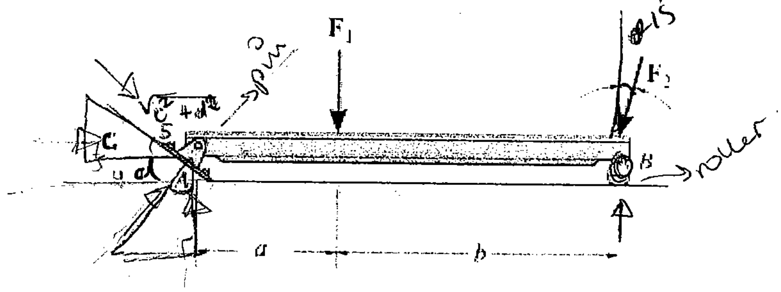
$\theta = 15°$

$a = 4$ m

$b = 8$ m

$c = 3$

$d = 4$



Problem 2

Determine the reactions at the supports.

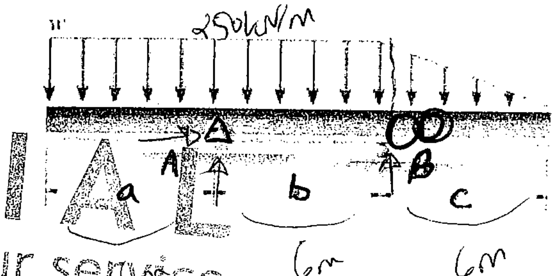
Given:

$w = 250$ kN/m

$a = 6$ m

$b = 6$ m

$c = 6$ m

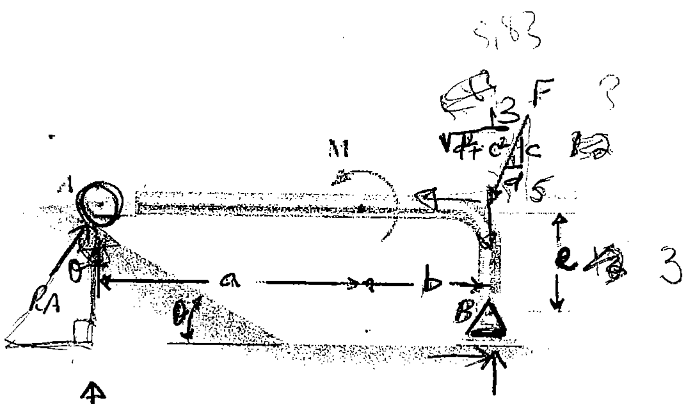


Problem 3

Determine the reactions at the roller *A* and pin *B*.

Given:

$M = 800$ kN.m	$c = 3$ m
$F = 390$ kN	$d = 5$
$a = 8$ m	$e = 12$
$b = 4$ m	$\theta = 30°$



Test:
3 problems - 4

- concept of FBD : equilibrium about particles
- equivalency (force summation , moment summation).
- compute reactions .

---

eng. research

Mechanics Homework 7

Problem 1:

$\xrightarrow{+}$ $\Sigma F_x = 0 \Rightarrow R_{Ax} - 400 \sin 15^{\circ} = 0$

$\Rightarrow R_{Ax} = 103.53$ $\quad (R_{Ax} = R_{Ax}' \frac{4}{5})$

$R_{Ax}' = \frac{5}{4} R_{Ax}$

$= \frac{5}{4} \cdot 103.53 = 129.4$ N

$\circlearrowleft + \quad \Sigma M_A = -600(4) + R_{By}(12) = 0$

$\Rightarrow R_{By} = 200$ N $\uparrow$

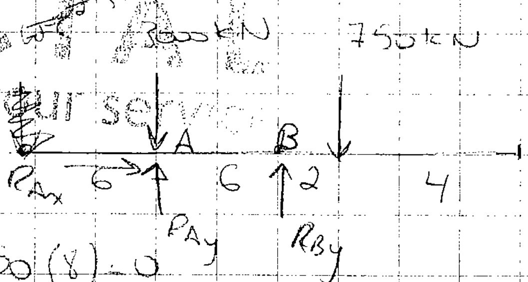
$+\uparrow \quad \Sigma F_y = R_{Ay} - 600 + 600 = 0$

$\Rightarrow R_{Ay} = 400$ N $\uparrow$

Problem 2:

$- 250 \times 12 = 3000$ kN

$- 250 \times 6/2 = 750$ kN



$\Rightarrow R_{Ax} = 0$

$\circlearrowleft + \quad \Sigma M_A = R_{By}(6) - 750(8) = 0$

$\Rightarrow R_{By} = 1000$ kN

$+\uparrow \quad \Sigma F_y = 0 \Rightarrow R_{Ay} - 3000 - 750 + 1000 = 0$

$\Rightarrow R_{Ay} = 2750$ kN $\uparrow$

Problem 3:

$\xrightarrow{+}$ $\Sigma F_x = R_{Bx} - 390\left(\frac{5}{5.83}\right) = 0 \Rightarrow R_{Bx} = 334.48$ kN $\rightarrow$

$\circlearrowleft + \quad \Sigma M_B = -800 + 390\left(\frac{5}{5.83}\right)(12) - R_{Ay}(12) = 0$

$\Rightarrow R_{Ay} = \cancel{334.48}$ 267.8 N

$\Rightarrow R_A = \frac{R_{Ay}}{\cos\theta} = \cancel{289.2}$ 309.24 kN

$+\uparrow \quad \Sigma F_y = R_{Ay} + R_{By} - 390\left(\frac{3}{5.83}\right) = 0 \Rightarrow R_{By} = -108.55$

309.24

$\Rightarrow R_{By} = 108.55$ kN

Problem 3

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow R_{Bx} - 390 \frac{5}{5.83} = 0$

$\Rightarrow R_{Bx} = 334.5 \text{ kN}$

$\overset{+}{\circlearrowleft} \Sigma M_A = 0 \Rightarrow -R_{Ay}(12) + 390 \frac{5}{5.83} \times 12 = 0$

$\Rightarrow R_{Ay} = \cancel{}\ 4013.76$

$\overset{+}{\uparrow} \Sigma F_y = 0 \Rightarrow R_{Ay} + R_{By} - 390 \left(\frac{3}{5.83}\right) = 0$

$\Rightarrow R_{Ay} = \cancel{-133.4 + \quad 133.4 \text{ kN}}$

$R_{By} = 200.47$

Problem 3

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow R_{Bx} - F\left(\frac{5}{13}\right) = 0$

$\Rightarrow R_{Bx} = 150 \text{ kN}$

$\overset{+}{\circlearrowleft} \Sigma M_A = 0 \Rightarrow -R_A \cos 30^\circ (12) + F\left(\frac{5}{13}\right)(3)$

$- R_A \sin 30^\circ (3) = 0$

$\Rightarrow -10.39 R_A - 1.5 R_A = -450$

$-11.89 R_A = -450$

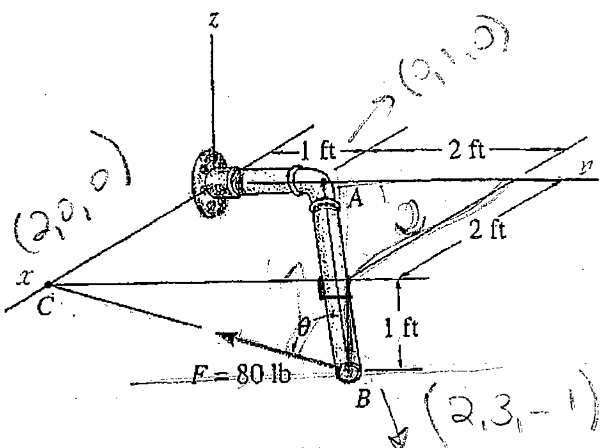
$R_A = 37.9 \text{ kN}$

$\overset{+}{\uparrow} \Sigma F_y = 0 \Rightarrow R_{By} - 390\left(\frac{12}{13}\right) + R_A \cos 30^\circ = 0$

$R_{By} = 327.06 \text{ kN}$

Problem 1

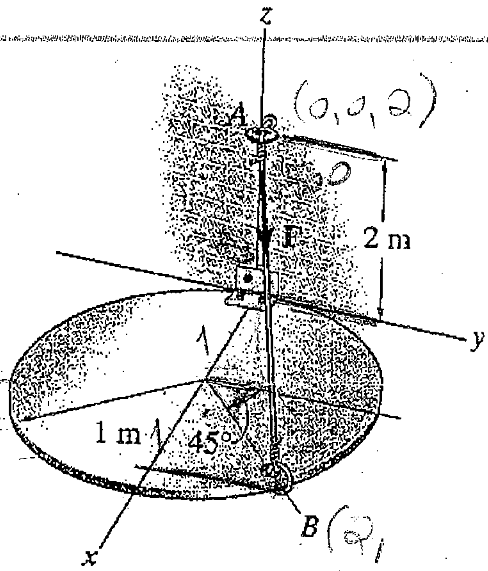
The pipe in figure (a) is subjected to the force of F = 80 lb. Determine the angle theta between **F** and the pipe segment BA, and the magnitudes of the component of **F**, which are parallel and perpendicular to BA.



(a)

Problem 2

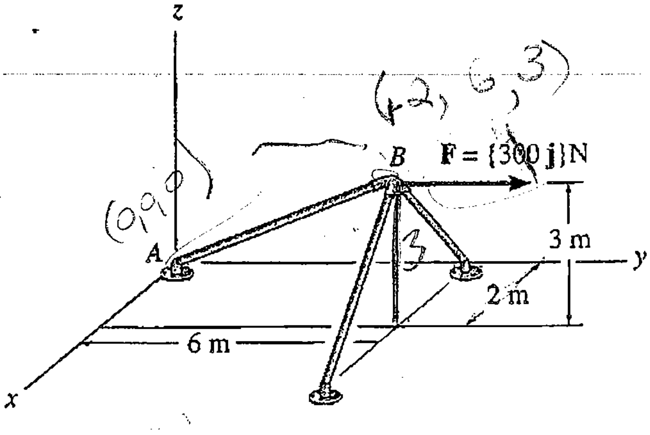
The circular plate in Figure (b) is partially supported by the cable AB. If the force of the cable on the hook at A is F = 500 N, express **F** as a Cartesian vector.



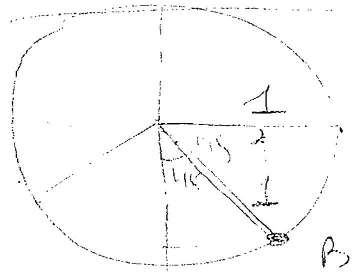
(b)

Problem 3

The frame shown in figure (c) is subjected to a horizontal force **F** = {300**j**} N. Determine the magnitude of the components of this force parallel and perpendicular to member AB

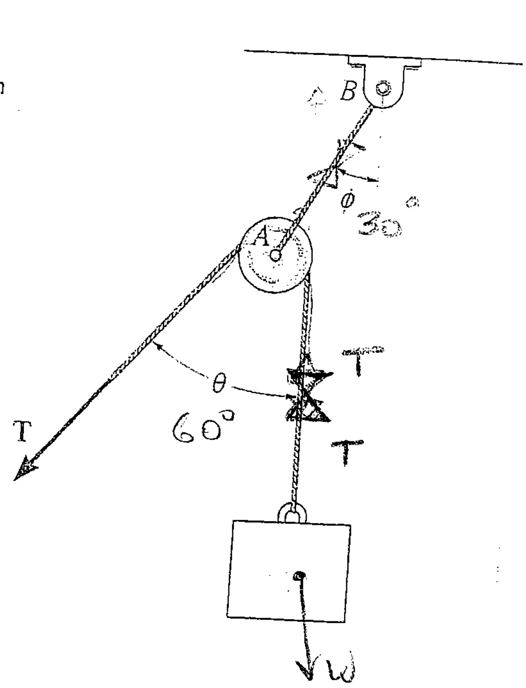


(c)



Classwork (1)

The block has weight W $\rightarrow 20\,N$ and is being hoisted at uniform velocity. Determine the angle θ for equilibrium and the required force in each cord.



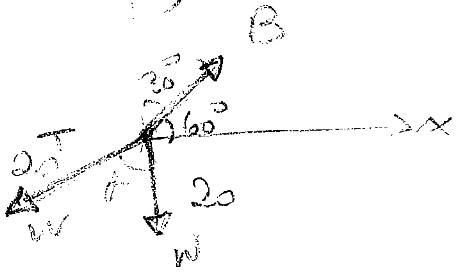
~~equilibrium $\sum F_y = 0$~~ $\uparrow(+)$

~~$T\cos\theta - W = 0$~~

~~$T\cos\theta = W$~~

~~$\cos\theta = \frac{W}{T}$~~

Isolate pulley



$\xrightarrow{+} \sum F_x = 0 \Rightarrow B\cos 60^\circ - 20\sin\theta = 0$

$+\uparrow \sum F_y = 0 \Rightarrow -20 \ldots B\cos 30^\circ = 0$

(1) $\sin\theta = \dfrac{B\cos 60^\circ}{-20}$

(2) $\cos\theta = \dfrac{+B\cos 30^\circ - 20}{20}$

$$\frac{0.25 B^2}{400} + \frac{0.75 B^2 + 400 - 34.6 B}{400} = 1$$

$$B^2 - 34.6 B = 0$$

$$B(B - 34.6) = 0$$

$\boxed{B = 34.6\,N}$ $\boxed{T = 20}$ $\boxed{W = 20}$

$$34.6\cos 60^\circ - 20\sin\theta = 0$$

$$20\sin\theta = 17.3$$

$$\sin\theta = 0.865$$

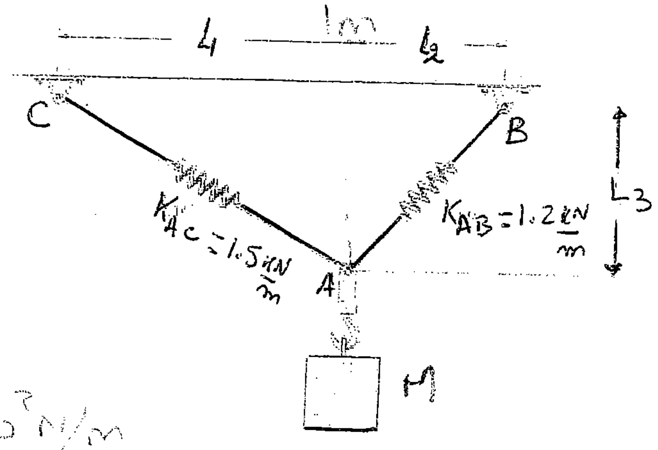
$$\theta = 60^\circ$$

Classwork (2) (* * *)

(1) The block of mass M is supported by two springs having the stiffness shown. Determine the unstretched length of each spring.

Units Used: $M = 30\,kg$ $L_1 = 0.6\,m$ $L_2 = 0.4\,m$ $L_3 = 0.5\,m$

$kN = 10^3\ N$



Let:

$M = 30\,kg$

$L_1 = 0.6\,m$

$L_2 = 0.4\,m$

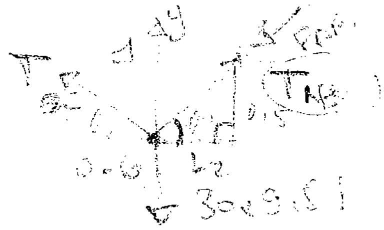
$L_3 = 0.5\,m$

$k_{AC} = 1.5\,kN/m = 1.5\times10^3\,N/m$

$k_{AB} = 1.2\,kN$

$g = 9.81\,m/s^2$

$W = 30\times9.81$



$\sqrt{0.4^2+0.5^2} = 0.64$

$\rightarrow \sum F_x = 0 \Rightarrow T_{AB}\cos\theta_1 - T_{AC}\cos\theta_2$ $\sqrt{0.6^2+0.5^2} = 0.78$

$T_{AB}\cdot\frac{0.4}{0.64} - T_{AC}\cdot\frac{0.6}{0.78} = 0$ ... (1)

$\uparrow \sum F_y = 0 \Rightarrow -W + T_{AB}\sin\theta_1 + T_{BC}\sin\theta_2$

$-294.3 + T_{AB}\frac{0.5}{0.64} + T_{AC}\frac{0.5}{0.78} = 0$

$0.625\,T_{AB} - 0.77\,T_{BC} = 0$

$0.78\,T_{AB} + 0.64\,T_{AC} = 294.3$

$T_{AB} = 226.48 = k(L - L_0) = 1.2\times10^3(0.64 - L_0)$

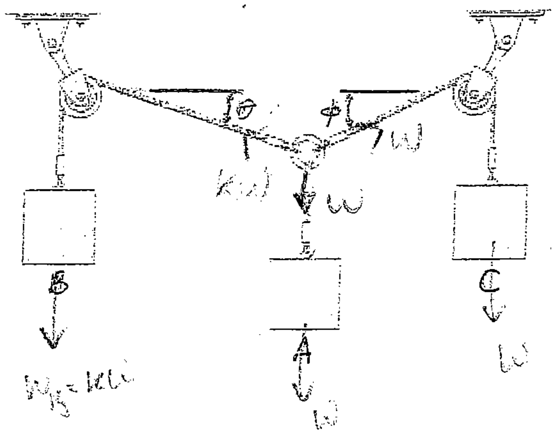
$\Rightarrow L_0 = 0.45\,m$

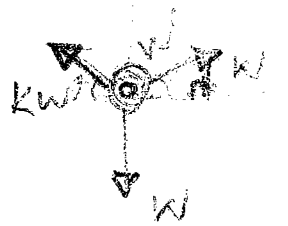
$T_{AC} = 183.8 = 1.5\times10^3(0.78 - L_0)$

$0.66\,m$

Classwork (3)

Three blocks are supported using the cords and two pulleys. If they have weights of $W_A = W_C = W$, $W_B = kW$, determine the angle θ for equilibrium.





in order to eliminate it when we square

$\cos\phi = k\cos\theta = 0$

$W\cos\phi - kW\cos\theta = 0$

$W\sin\phi + kW\sin\theta$

$W\cos\phi - kW\cos\theta = 0$

$W\sin\phi + kW\sin\theta = W$

$\cos\phi - \cos\theta = 0$

$\cos\phi = \cos\theta$

$\theta = \sin^{-1}(k/2)$

$\cos^2\phi = k^2\cos^2\theta$

$\sin^2\phi = (1 - k\sin\theta)^2$

$\cos^2\phi + \sin^2\phi = k^2\cos^2\theta + 1 + k^2\sin^2\theta - 2k\sin\theta$

$k^2(\cos^2\theta + \sin^2\theta) - 2k\sin\theta = 0$

$k^2 - 2k\sin\theta = 0$

$k - 2\sin\theta = 0$

$\sin\theta = k/2$

$\theta = \sin^{-1}\frac{k}{2}$

If it takes a force **F** to pull the nail out, determine the smallest vertical force **P** that must be applied to the handle of the crowbar. *Hint:* This requires the moment of **F** about point *A* to be equal to the moment of **P** about *A*. Why?

Given:

$F = 125$ lb

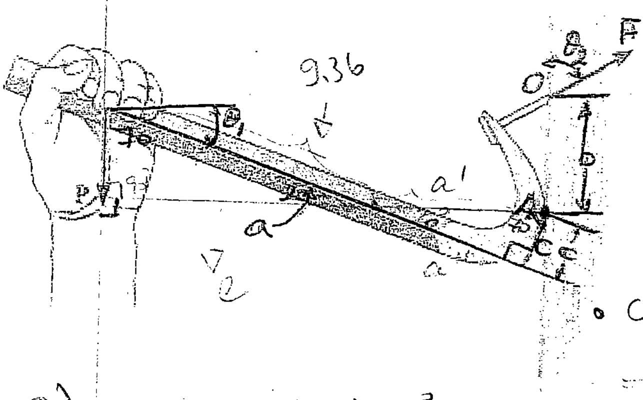
$a = 14$ in

$b = 3$ in

$c = 1.5$ in

$\theta_1 = 20$ deg

$\theta_2 = 60$ deg



• $\cos 70^\circ = \frac{c}{a'}$

$\Rightarrow a' = 4.39$ in

$a'' = \sqrt{a'^2 + \ldots}$

$= 4.64$ in

• $\cos 20^\circ = \frac{c}{9.36}$

$\Rightarrow c = 4.16$ in

$M_F = -(125 \sin 60^\circ \times 3) = -324.76$ lb.in

$|-324.76| = |M_P|$

$\Rightarrow |-324.76| = |\vec{F}_P| \times 13.19$

$\Rightarrow |F_P| = 24.66$ lb

Why?

equal & opp. moments at pt A ⇒ pt A is static & doesn't move, fixing the crowbar on the wall, thus allowing ~~the~~ it to pull the nail.

The force **F** acts on the end of the pipe at B. Determine (a) the moment of this force about point A, and (b) the magnitude and direction of a horizantal force, applied at C, which produces the same moment.

Given:

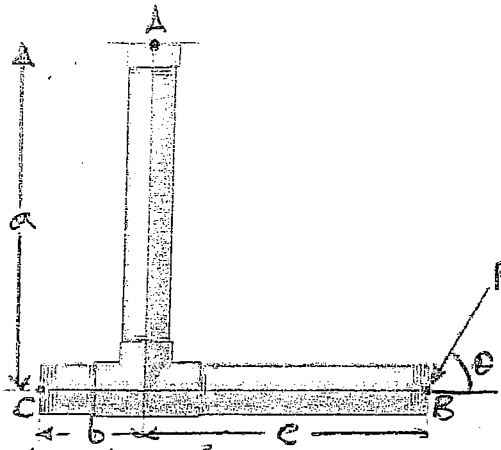
$F = 70$ N

$a = 0.9$ m

$b = 0.3$ m

$c = 0.7$ m

$\theta = 60$ deg



$M = -(70 \cos 60^\circ \times 0.9) - (70 \sin 60^\circ \times 0.7)$

$= -73.94$ N m $= 73.94$ Nm

$M = C \times 0.9$

$\Rightarrow C = -73.94$ Nm $\div 0.9$

$\Rightarrow |C| = \cancel{82.16\,N}$ 82.16

in the -ve x-axis

Given: Find M_A??

$F = 52$ lb

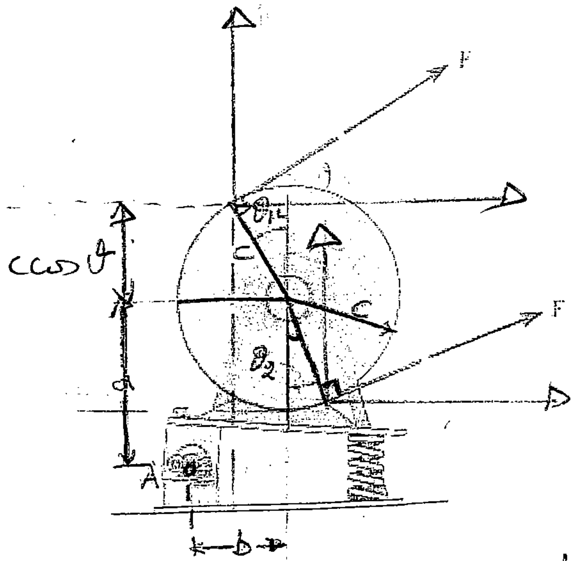
$a = 8$ in

$b = 5$ in

$c = 6$ in

$\theta_1 = 30$ deg

$\theta_2 = 20$ deg



$$M_A = -\left(52\cos 30^\circ\right)\overset{a+c\cos\theta_1}{(13.2)} + \left(52\sin 30^\circ\right)\overset{b-c\sin\theta_1}{\left(5-6\sin 30^\circ\right)}$$

$$-\left(52\cos 20^\circ\right)\overset{a-c\cos\theta_2}{(2.36)} + \left(52\sin 20^\circ\right)\overset{b+c\sin\theta_2}{(7)}$$

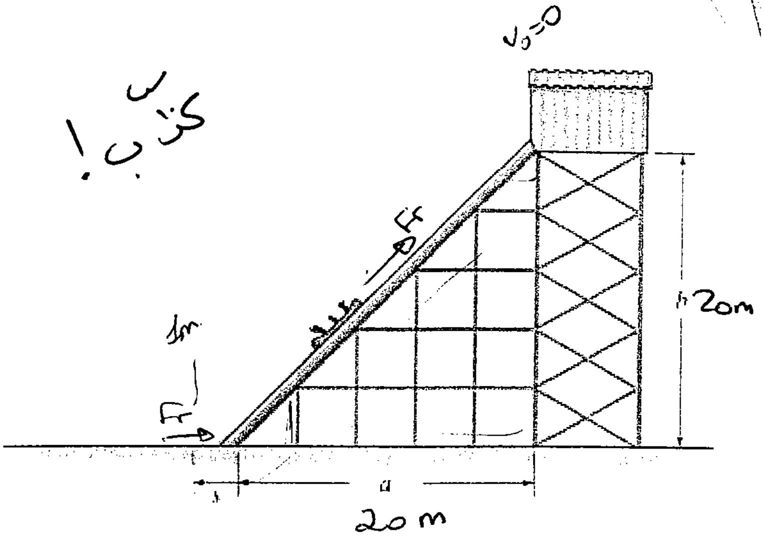
$$= -594.44 + 52 - 115.32 + 124.5$$

$$= -533.26 \text{ lb in} = 533.26$$

Kinetics (Newton's Law)

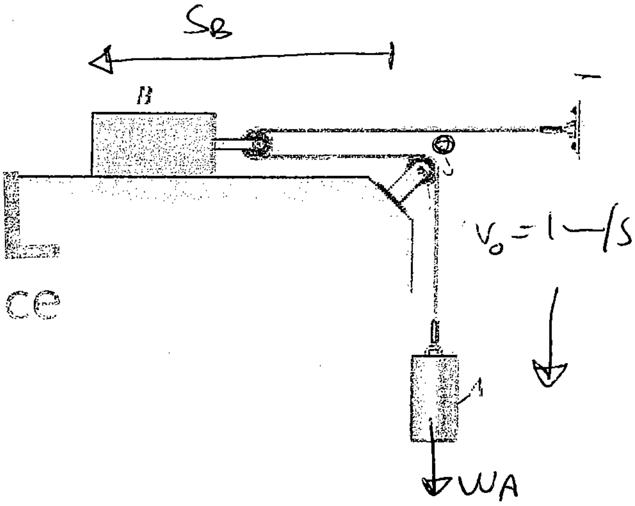
Problem 1

The water-park ride consists of a sled of weight W =8-kN (≈800-kg) which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is F_r = 300N and in the pool for a short distance is F_r = 800N, determine how fast the sled is traveling when s = 1m (Ans:V_2=19.2 m/s)



Problem 2

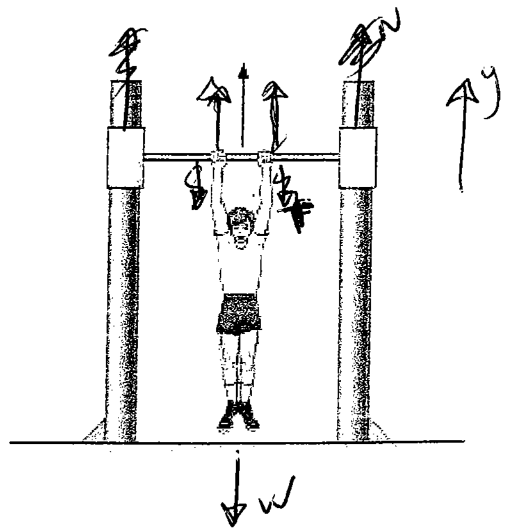
At a given instant block A of weight W_A = 25N is moving downward with a speed V_1 = 1m/s at t=0. Determine its speed at the later time t = 2s. Block B has weight W_B = 30N, and the coefficient of kinetic friction between it and the horizontal plane is μ_k=0.3. Neglect the mass of the pulleys and cord. (Ans: V_A=13.4 m/s)



Problem 3

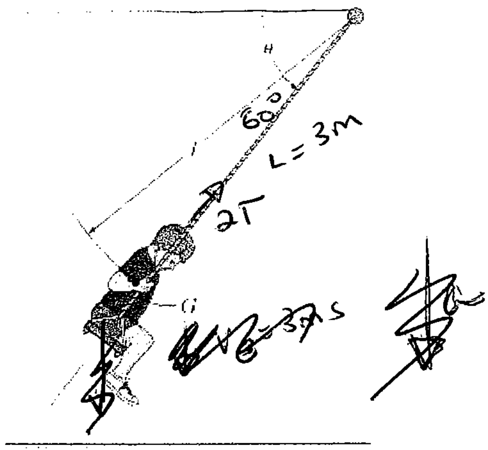
The boy has weight W = 400N (≈ 40kg) and hangs uniformly from the bar. Determine the force in each of his arms at time t =2s if the bar is moving upward with (a) a constant velocity of 1m/s and (b) a speed v = $1.2t^2$ m/s.
(Ans: (a) T=200N; (b) T=298N)

equilibrium



Problem 4

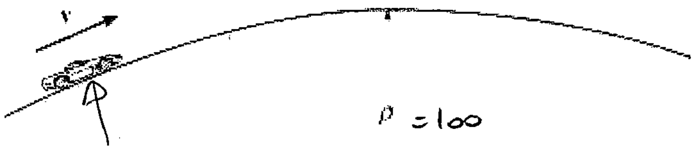
At the instant $\theta = \theta_1$ the boy's center of mass G has a downward speed $V_G = 3$m/s. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight W = 300N (≈30Kg). Neglect his size and the mass of the seat and cords.
(Ans: T=175.8N , a_t= 4.91 m/s^2)



$\Rightarrow a_t = 0 \Rightarrow a = a_n$

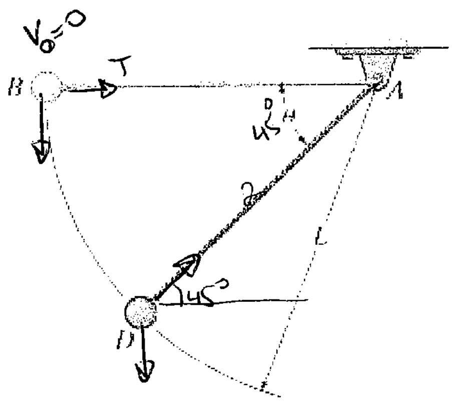
Problem 5

If the crest of the hill has a radius of ~~curvature~~ curvature ρ, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has weight W.
(Ans: V =31.3 m/s = 111.6 km/h)



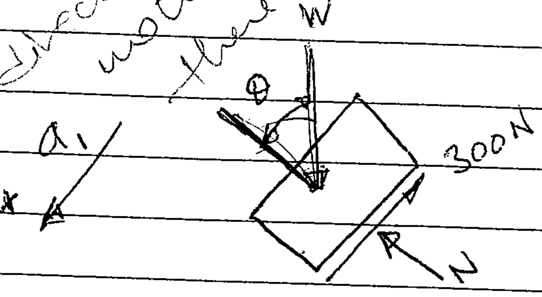
Problem 6

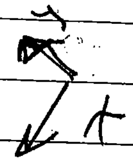
The pendulum bob *B* of mass *M* = 5 kg is released from rest when θ = 0°. Determine the initial tension in the cord and also at the instant the bob reaches point *D*, θ = 45°.Given: L = 2m. Neglect the size of the bob. (Ans: T= 0N; T_D = 104.1 N)

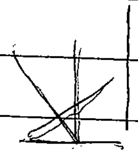


Take the direction (i.e. x-axis is) as direction of motion when there is no fixed support

Problem 1







$$\Sigma F = m a_1$$

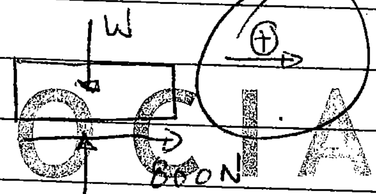
$$W\sin\theta - 300 = 800\, a_1$$

$$a_1 = [8000\sin 45 - 300]/800 \Rightarrow a_1 = 6.69\ m/s^2$$

Kinematics: $S_1 = \sqrt{a^2+b^2} = 28.28\ m$

$$v_1^2 = 2a_1 S_1 = 2(6.69)(28.28) = 19.45\ m/s$$

In water: ←



like motion

$-800 = ma$
$\Rightarrow a = -1\ m/s$
$= 1 \leftarrow s$

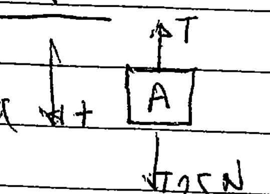
$$800 = m a_2$$

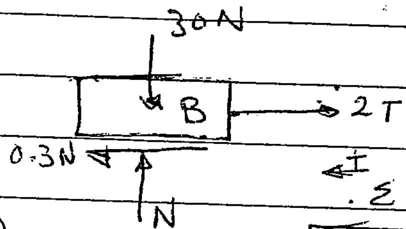
$$a_2 = \frac{800}{800} = 1\ m/s^2 \rightarrow$$

$$v_2^2 - v_1^2 = 2a_2 S_2$$

$$v_2^2 = (19.45)^2 + 2(1)(1) \Rightarrow v_2 = 19.2\ m/s$$

Problem 2



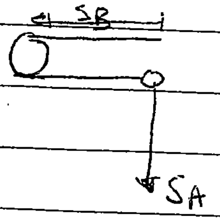


$$\boxed{25 - T = 2.5\, a_A} \quad (1)$$

$\overset{+}{\leftarrow}\ \Sigma F_x = m a_B$

$$\boxed{-2T + 0.3N = 3\, a_B} \quad (2)$$

$$\Sigma F_y = 0 \Rightarrow N = 30\ N$$

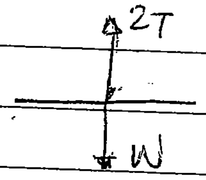


$$S_A + 2S_B = L \Rightarrow \boxed{a_A = -2a_B} \quad (3)$$

Solve (1), (2), and (3) $\quad a_A = 6.30\ m/s^2$

$$v = v_0 + at = 1 + (6.30)(2) = 13.4\ m/s$$

Problem 3



① $2T - W = 0 \Rightarrow T = \frac{W}{2} = 200\,N$

② $2T - W = ma \qquad a = \frac{dv}{dt} = \frac{d}{dt}(1.2t^2) = 2.4t$

$t = 2s \quad a = 4.8\,m/s^2$

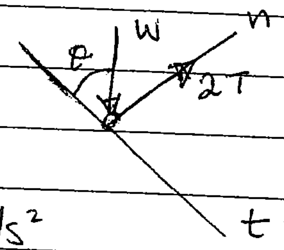
$2T - 400 = (40)(4.8)$

$T = 296\,N$

Problem 4

$\Sigma F_t = m a_t$

$mg\cos\theta = m a_t$



$a_t = g\cos\theta = g\cos 60 = 4.905\,m/s^2$

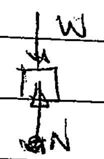
$\Sigma F_n = m a_n$

$2T - mg\sin 60 = m\frac{v^2}{\rho}$

$2T = 300\sin 60 + \frac{(30)(9)}{3} \Rightarrow T = 175\,N$

Problem 5

Limiting Case $N = 0$ 2



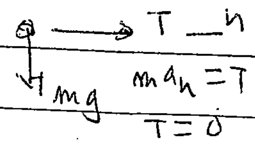
$\Sigma F_n = m a_n$

$mg = m\frac{v^2}{\rho} \Rightarrow v = \sqrt{\rho g} = \sqrt{100 \times 10} \simeq 31.3\,m/s$

Problem 6

Initial Tension : $T = 0$, $v = 0$, $a_n = 0$

Point D :

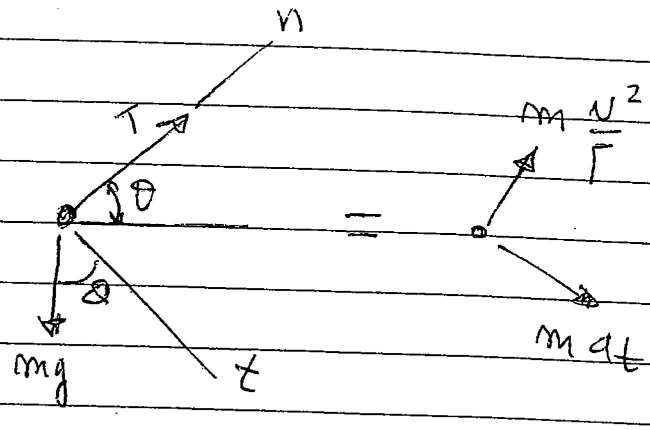


$m a_n = T$

$T = 0$

$\Sigma F_n = m a_n$

$T - mg \sin 45 = m \dfrac{v^2}{\rho}$



$\Sigma F_t = m a_t$

$mg \cos 45 = m a_t \Rightarrow a_t = g \cos 45 = 6.937\ m/s^2$

Having:

$v \dfrac{dv}{ds} = g \cos\theta \Rightarrow \int_0^v v\,dv = \int_0^{45°} (g \cos\theta)\, 2\, d\theta \;;\; ds = \rho d\theta$

$\dfrac{v^2}{2} = 2g \sin\theta \Big|_0^{45} \Rightarrow v = 5.26\ m/s$

$T = \cancel{m} g \sin 45 + \cancel{m} \dfrac{(5.26)^2}{2} = 104\ N$

$a_t ds = v dv$

$a_t = g \cos\theta \qquad ds = \rho d\theta$

$\Rightarrow \int_0^{45°} g \cos\theta \cdot \rho\, d\theta = \int_0^v v\,du$

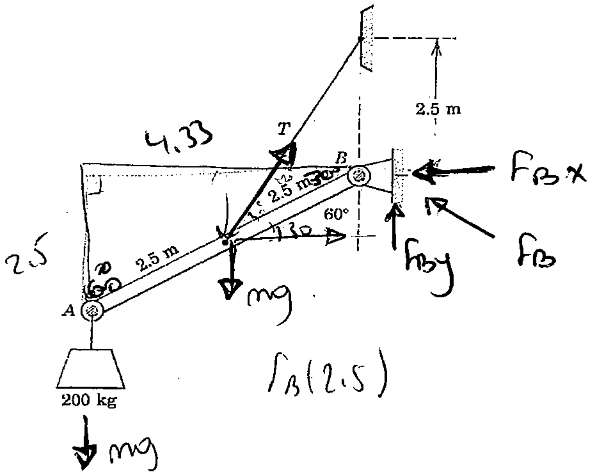
ENG 212 December 21, 2009
Test 2

Problem 1 30 Points

The uniform bar AB has mass 50 kg and supports the 200-kg load at A. Calculate the tension in the cable and the magnitude F_B of the force supported by the pin at B.

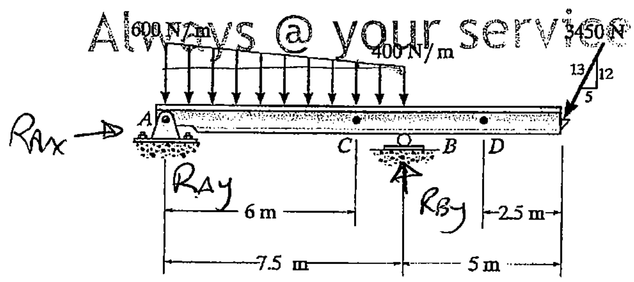
$\Sigma F_x = 0$

$\Sigma F_y = 0$



Problem 2 45 Points

Determine the internal normal force, shear force and moment at points C and D of the beam.



$T\cos\theta = 0$

$F_A\cos\theta - W + T\sin\theta + F_B\cos\theta = 0$

$T\cos\theta - F_B\sin\theta - F_A\sin\phi = 0$

Problem 3 25 Points

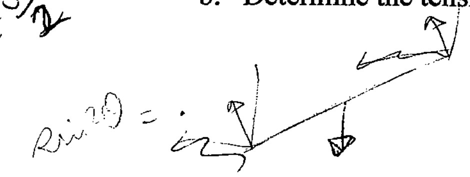
The man has weight W and stands at the center of the plank. If the plane at A and B are smooth (reactions have only normal components).

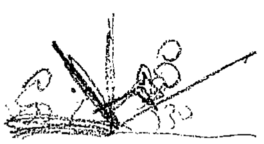
a. Draw free body diagram of the plank AB.
b. Determine the tension in the cord in terms of W and θ.

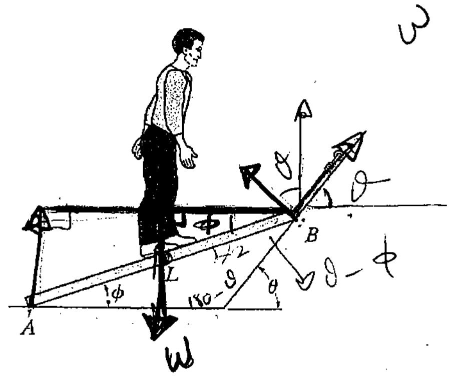
$T = \frac{W}{2}\sin\theta$

$F_B\sin\theta + F_A\sin\phi$

$F_B\cos\theta + F_A\cos\phi$







$= 0$

$F_B(90 - \theta)$

$\Sigma F_y = 0$

$F_A - W + T\sin\theta + F_B\sin(90° - \theta) = 0$

$\frac{T\cos\theta^2}{F_B^2} + \frac{(-F_A + W - T\sin\theta)}{F_B^2}$

$T\cos\theta =$

$F_B\cos(90 - \theta) = T\cos\theta$

$F_B\sin(90 - \theta) = -F_A + W - T\sin\theta$

$180 - (\phi + 180 - \theta)$

$$R_A - W + T\sin\theta + R_B\cos\theta = 0$$

$$R_B\sin\theta + T\cos\theta = 0$$

$$T\sin\theta = -R_B\cos\theta + W - R_A$$

$$\boxed{T\cos\theta = R_B\sin\theta}$$

$$\frac{}{T} \qquad T\cot\theta$$

$$T^2 = R_B\sin\theta + ($$

$$\tan\theta = \frac{T}{R_B}$$

$$R_A - W + T\sin\theta + R_B\left(\frac{R_B\sin\theta}{T}\right)$$

$$R_A - W + T\sin\theta + \frac{R_B^2}{T}\sin\theta = 0$$

$$\sin\theta\left(T + \frac{R_B^2}{T}\right) = W - R_A$$

$$\sin\theta\left(\frac{T^2 + R_B^2}{T}\right) = W - R_A$$

$$\Sigma M_B = 0 \Rightarrow W\left(\tfrac{L}{2}\cos\phi\right) - R_A(L\cos\phi) = 0$$

$$W \cdot \frac{L}{2}\cancel{\cos\phi} = R_A \cdot L\cancel{\cos\phi}$$

$$R_A = \frac{W}{2}$$

$$\frac{T}{\sin\theta} = \frac{W}{2}$$

$$\boxed{T = \frac{W}{2}\sin\theta}$$

$$\frac{W}{2} - W + T\sin\theta + T\cot\theta\cos\theta = 0$$

$$-\frac{W}{2} + T\left(\sin\theta + \frac{\cos^2}{\sin\theta}\right) = 0$$